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**NETWORK THEORY** 

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# **GATE 2025**

# NETWORK THEORY (ECE – EE – IN)

#### GATE ECE 2025 - Network theory - Syllabus

Circuit analysis: Node and mesh analysis, superposition, Thevenin's theorem, Norton's theorem, reciprocity.

Sinusoidal steady state analysis: phasors, complex power, maximum power transfer.

Time and frequency domain analysis of linear circuits: RL, RC and RLC circuits, solution of network equations using Laplace transform.

Linear 2-port network parameters, wye-delta transformation.

#### GATE EE 2025 - Network theory - Syllabus

Network elements: ideal voltage and current sources, dependent sources, R, L, C, M elements; Network solution methods: KCL, KVL, Node and Mesh analysis; Network Theorems: Thevenin's, Norton's, Superposition and Maximum Power Transfer theorem; Transient response of dc and ac networks,

Sinusoidal steady-state analysis, resonance, complex power and power factor in ac circuits.

balanced three phase circuits, star-delta transformation,

Two port networks.

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### **NETWORK THEORY**

## BASICS - DC ANALYSIS AND NETWORK THEOREMS

**THEORY – SHORT NOTES** 

#### CHAPTER 1 BASICS – NETWORK THEOREMS

#### **TOPIC 1** $\rightarrow$ **BASIC TERMS**

Voltage is a consequence of accumulation of charges Potential is the ability of a charge to do work Potential at a point is the energy per unit charge

**V** (Volts) = 
$$\frac{W(Joules)}{Q(Coloumbs)}$$

Potential difference or Electromotive force EMF is the cause of current

Current is the flow of charges.

I (Ampere) =  $\frac{Q (Coloumbs)}{t (seconds)}$ 

Power is rate of change of energy per unit time. Power is said to flow when there is both voltage and current flowing.

**Power P (Watts)** =  $\frac{W (Joules)}{Q (Coloumbs)}$  **X**  $\frac{Q (Coloumbs)}{t (seconds)}$  =  $\frac{W (Joules)}{t (seconds)}$ 

#### TOPIC 1.1 $\rightarrow$ Kirchoff's Laws

1. Voltage Law  $\rightarrow$  (KVL)

The sum of all the voltages in any closed loop has to be zero since voltage is energy and it is conserved

2. Current Law  $\rightarrow$  (KCL)

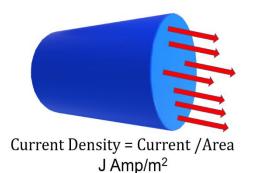
The sum of all the currents entering or leaving a node has to be zero since current is charge and it is conserved

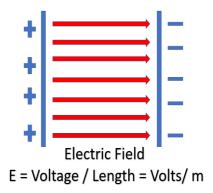
#### TOPIC 1.2 $\rightarrow$ Ohms Law

The current in a thin wire is proportional to the voltage applied across it's ends.

 $I \alpha V \rightarrow V = I R$ 

The current density in a thick conductor of finite cross section area is proportional to the electric field across its ends.





 $\mathbf{J} \propto \mathbf{E} \rightarrow \mathbf{J} = \mathbf{\sigma} \mathbf{E} ,$ 

Where  $\sigma$  = conductivity of the material = mho / m

#### TOPIC 2 $\rightarrow$ SERIES and SHUNT

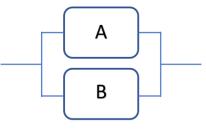
When a 2 terminal device is connected to another 2 terminal device with only one terminal in common, it is series



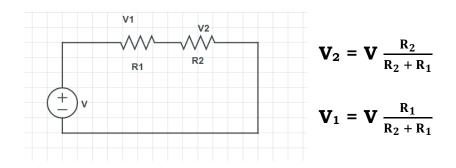
In series current is same, voltage is different across each element

When a 2 terminal device is connected to another 2 terminal device with both the terminals in common, it is shunt or parallel

In shunt voltage is same, current is different in each element



Voltage Division rule in series resistors



#### **Current Division rule in shunt resistors**



**Resistors in series and Equivalent resistance** 

$$\mathbf{Req} = \mathbf{R}_1 + \mathbf{R}_2$$

**Resistors in shunt and Equivalent resistance** 

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

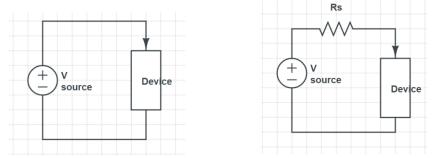
A short circuit parallel to any element is equal to short circuit. An open circuit parallel to any element is equal to the element itself

Two ideal current sources cannot be connected in series Two ideal voltage sources cannot be connected in parallel

Voltage is measured in parallel but added in series Current is measured in series but added in parallel

#### **TOPIC 3** $\rightarrow$ **VOLTAGE SOURCE and CURRENT SOURCE**

If the voltage across it's terminals is fixed for any load or current drawn from the source, it is called as ideal voltage source. Ex: 220V power supply sockets in home

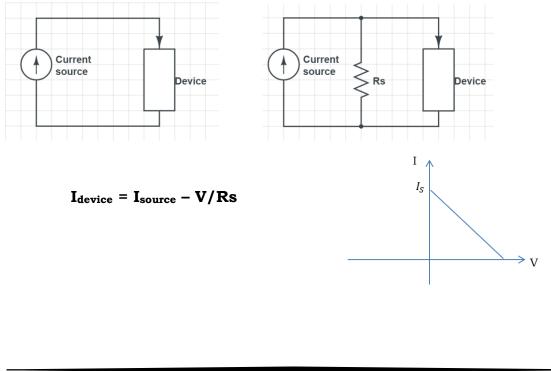


If the voltage drops with increasing current it is a non-ideal source.



If the current through it's terminals is fixed for any load , it is called as ideal current source.

If the current drops with increasing load it is a non-ideal source.



#### **NETWORK THEORY**

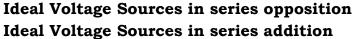
Ideal sources can deliver unlimited power, which depends on load.

Non ideal sources can deliver a finite power whose value ranges  $u^2$ 

from zero (minimum) to a maximum of  $\frac{V^2}{R_S}$  or I<sup>2</sup> R<sub>s</sub>.

#### **TOPIC 3.1** $\rightarrow$ Addition and Subtraction of Sources

Voltage is measured in shunt and added in series



Current is measured in series but adds in shunt

Ideal Current Sources in shunt addition Ideal Voltage Sources in shunt opposition

Two unequal ideal voltage sources cannot be connected in parallel Two unequal ideal current sources cannot be connected in series **TOPIC 3.2**  $\rightarrow$  Power absorbed and Power delivered by the source

1. If current enters into the positive terminal of source then it is referred as absorbed power

2. If current leaves from positive terminal of voltage source then it is referred as delivered power

**TOPIC 3.3**  $\rightarrow$  Dependent and Independent sources

If the voltages or currents depend on voltages or current at a different point, they are said to be dependent sources.

Ex: Current in a BJT collector depends on base current. Ex: Voltage across a diode is dependent on the external bias voltage.

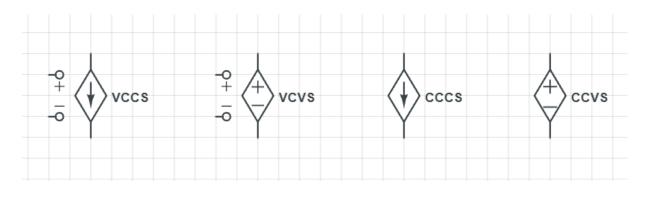
Dependent sources can be of 4 types.

Current dependent Current source

**Current dependent Voltage source** 

Voltage dependent Current source

Voltage dependent Voltage source



#### **TOPIC 4** $\rightarrow$ **CLASSIFICATON OF CIRCUIT ELEMENTS**

**TOPIC 4.1**  $\rightarrow$  **LINEAR and NON-LINEAR ELEMENTS** 

If the current flow in an element is directly proportional to the device voltage, the element is called as Linear element

The device obeys Ohm's Law

Example  $\rightarrow$  Resistors (In DC and AC both)

Inductors and Capacitors( In AC only)

If the output in an element is directly proportional to the input

parameter of voltage or current , this element is also called as Linear

element. The device may or may not obeys Ohm's Law

Example  $\rightarrow$  BJT in active region , Diode in forward bias condition.

#### **TOPIC 4.2** $\rightarrow$ **ACTIVE and PASSIVE ELEMENTS**

When the source or element delivers power it is called as Active element.

Example  $\rightarrow$  A voltage or current source

When the source or element absorbs power it is called as Passive element.

Example  $\rightarrow$  Resistors, diodes, electronic components.

#### **TOPIC 5** $\rightarrow$ **RESISTOR AND IT'S IMPORTANCE**

Resistance is the slope of transformation for a given voltage(V) and the produced current(I)

This slope of transformation ( resistance) depends on the properties or physical conditions.

$$\mathbf{R} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\rho \, \mathbf{L}}{\mathbf{A}}$$

Where  $\rho$  = resistivity of the material =  $\frac{1}{2}$ 

 $\sigma$  = conductivity of the material

Resistance is the cause of power dissipation

Ex: speakers, lights, heater are called as loads or resistances

#### **TOPIC 6** $\rightarrow$ **INDUCTOR and CAPACITOR**

Inductance is the slope of transformation for a given current (I) and the produced magnetic Flux (  $\phi$  ) by this current.

This slope of transformation depends on the properties or physical conditions like winding turns, length d and area A

$$\mathbf{L} = \frac{\mathbf{\phi}}{\mathbf{I}} = \frac{\mathbf{N}^2 \boldsymbol{\mu} \mathbf{A}}{\mathbf{d}}$$

According to Faraday's Law

Rate of change of magnetic flux with time is voltage (EMF)

$$\frac{d\phi}{dt} = \mathbf{V} \text{ and } \mathbf{L} = \frac{d\phi}{dI} \times \frac{dt}{dt} = \mathbf{V} \frac{dt}{dI}$$
$$\mathbf{V} = \mathbf{L} \frac{dI}{dt}$$

Solving the above equation,

 $I_o$  is the initial current

 $\mathbf{I} = \mathbf{I}_0 + \frac{1}{L} \int \mathbf{V}(t) dt$ 

Capacitance is the slope of transformation for a given voltage(V) and the produced electric charge( Q) by this voltage.

This slope of transformation depends on the properties or physical conditions like length and area.

$$\mathbf{C} = \frac{\mathbf{Q}}{\mathbf{V}} = \frac{\varepsilon \mathbf{A}}{\mathbf{d}}$$
$$\mathbf{C} = \frac{\mathbf{dQ}}{\mathbf{dV}} \times \frac{\mathbf{dt}}{\mathbf{dt}} = \mathbf{I} \frac{\mathbf{dt}}{\mathbf{dV}}$$
$$\mathbf{I} = \mathbf{C} \frac{\mathbf{dV}}{\mathbf{dt}}$$

Solving the above equation ,  $V = V_0 + \frac{1}{c} \int I(t)dt$ Vo is the initial voltage

Resistance and Inductances add in series but Capacitance adds in parallel

Series Inductors  $L_{eq} = L_1 + L_2$ 

Shunt Capacitors  $C_{eq} = C_1 + C_2$ 

Series Capacitors  $C_{eq} = \frac{C_1 C_2}{C_1 + s C_2}$ 

**Shunt Inductors**  $L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$ 

#### TOPIC 7 $\rightarrow$ DC and AC voltages

DC voltage stands for Direct current voltage

DC voltage has a constant value at any time.

DC current has unidirectional flow of electrons at a constant velocity.

AC voltage stands for Alternating current voltage

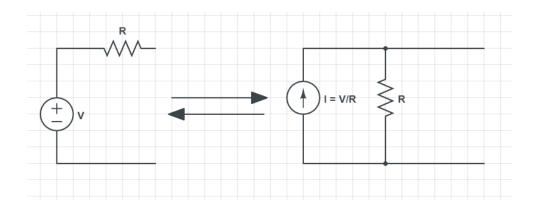
The voltage or current changes its polarity and hence the direction of moving electrons changes periodically with time

#### **TOPIC 8** $\rightarrow$ **CIRCUIT REDUCTION TECHNIQUES**

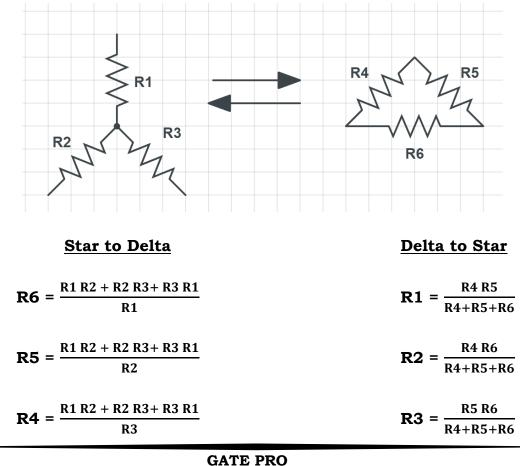
#### **TOPIC 8.1** $\rightarrow$ **SOURCE TRANSFORMATION**

Any Voltage source with a series resistance can be replaced with a current source and shunt resistance.

The vice-versa is also true that the current source can be replaced with voltage source.



TOPIC 8.2  $\rightarrow$  STAR - DELTA TRANSFORMATION

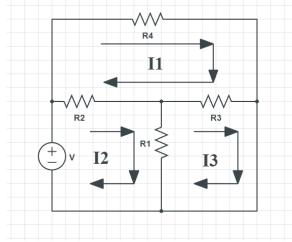


#### TOPIC 8.3 $\rightarrow$ MESH ANALYSIS

A mesh is a combination of visible closed loops in the given circuit. Each loop is assigned a current called as mesh current and the KVL is written for each loop.

The elements common to two loops are deemed to have the currents as sum or difference of the mesh current.

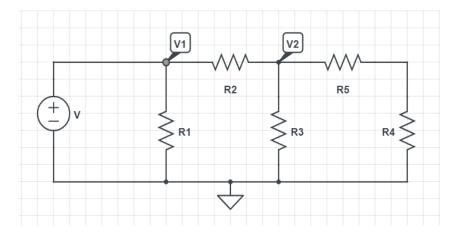
The solution of the KVL gives all voltages and currents in the circuit.



TOPIC 8.4  $\rightarrow$  NODAL ANALYSIS

A node is a junction of elements in the given circuit. Each node is assigned a voltage and the KCL is written at each node

The solutions of the KCL equations gives all voltages and currents.

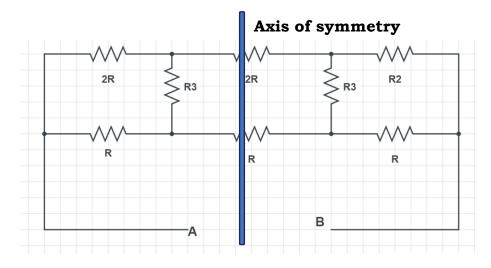


#### **TOPIC 8.5** $\rightarrow$ **SYMMETRY IN A NETWORK**

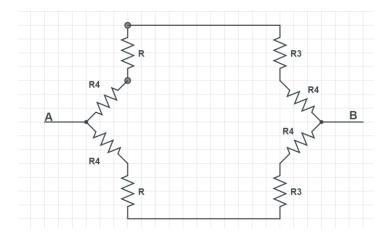
If two points of a network are identically located with respect to each of the terminals then they are said to be equipotential and these points can be short circuited or open circuited according to current flow conditions.

#### Mirror Symmetry or Vertical Symmetry

Elements in the network are symmetric and overlap on each other when the input terminals or across terminals A and B overlap on each other.



Folding Symmetry or Horizontal Symmetry



The line of symmetry is the line joining A and B.

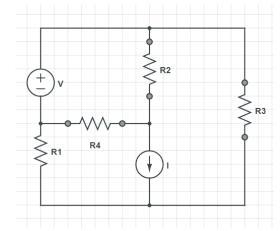
#### **TOPIC 9** $\rightarrow$ **NETWORK THEOREMS**

#### **TOPIC 9.1** $\rightarrow$ **SUPER-POSITION THEOREM**

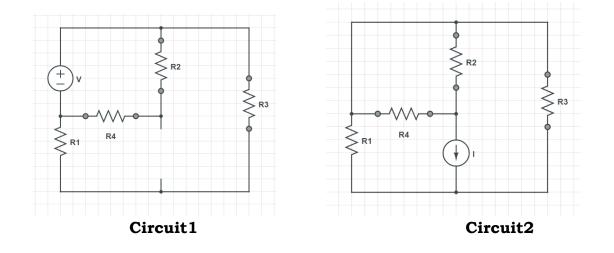
In any linear and bi-directional circuit having multiple sources or active elements, the current or voltage in any branch can be calculated as the algebraic sum of current or voltage in that branch considering one source at a time and replacing other sources with their internal resistances.

The theorem is applied by replacing the voltage source with short circuit (V =0) and current source with open circuit (I = 0)

In any of the branches shown below  $I = I_1 + I_2$ 



Where  $I_1$  flows in circuit1 and  $I_2$  flows in circuit2



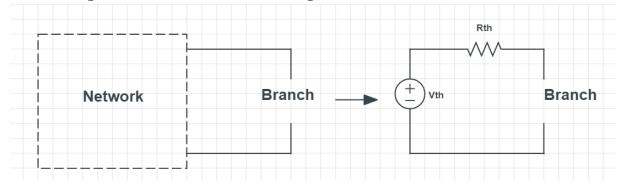
#### TOPIC 9.2 $\rightarrow$ THEVENIN'S THEOREM

Any linear and bi-directional circuit having multiple sources or active elements, the entire network can be replaced with a voltage source and series resistance.

The voltage source value is the open circuit voltage across the branch.

The series resistance is the equivalent resistance across the branch.

Vth = Voc across the branch in the presence of the network Rth = Req across the branch in the presence of the network

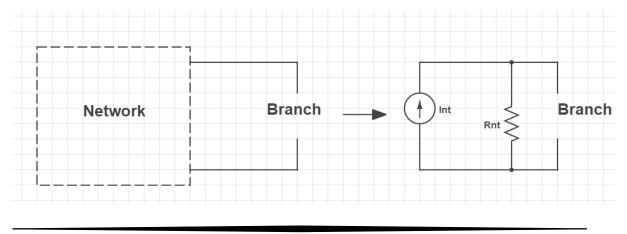


#### TOPIC 9.3 $\rightarrow$ NORTON'S THEOREM

Any linear and bi-directional circuit having multiple sources or active elements, the entire network can be replaced with a current source and shunt resistance.

The current source value is the short circuit current in the branch. The shunt resistance is the equivalent resistance across the branch.

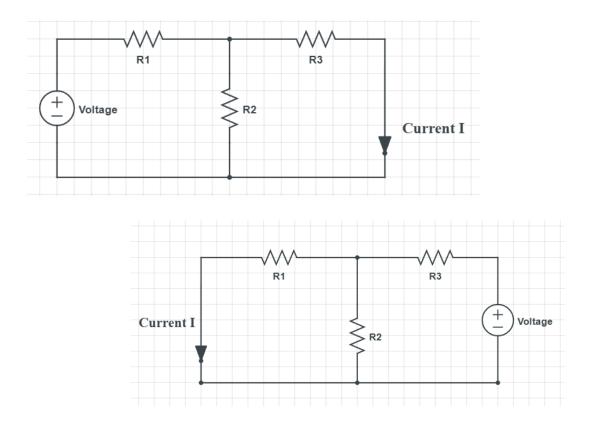
 $I_{NT}$  = Isc through the branch in the presence of the network  $R_{NT}$  = Req across the branch in the presence of the network



#### TOPIC 9.4 $\rightarrow$ RECIPROCITY THEOREM

It states that the current at one point in a circuit due to a voltage at a second point is the same as the current at the second point due to the same voltage at the first.

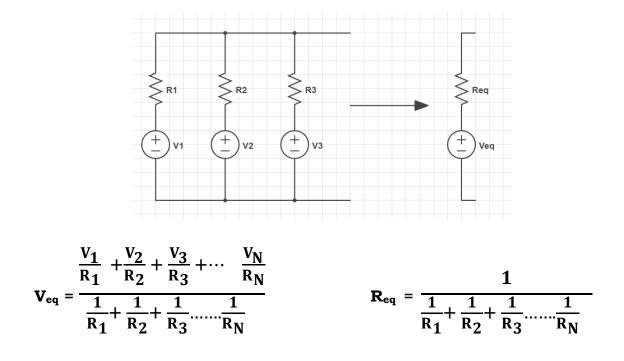
The ratio of V/I at the cause and effect remains the same in spite of interchanging cause and effect positions.



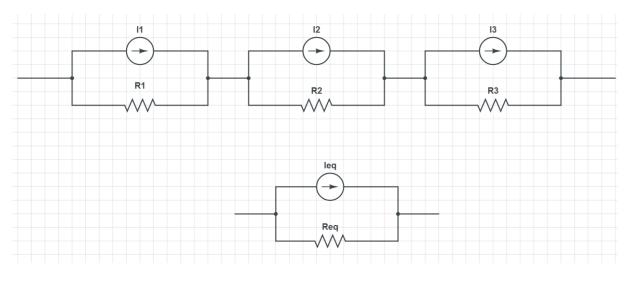
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#### TOPIC 9.5 $\rightarrow$ MILLMAN'S THEOREM

When N non ideal voltage sources are connected in parallel, the equivalent non ideal source can be written as



When N non ideal voltage sources are connected in series the equivalent non ideal source can be written as

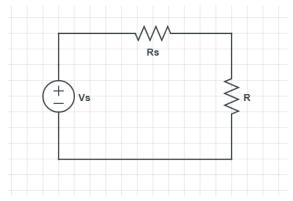


<b>.</b> _	$\underline{I_1 R_1 + I_2 R_2 + \cdots \dots I_N R_N}$
I <sub>eq</sub> =	$R_1 + R_2 + \cdots = R_N$

 $\mathbf{R}_{eq} = \mathbf{R}_1 + \mathbf{R}_2 + \cdots \dots \mathbf{R}_N$ 

#### **TOPIC 9.6** $\rightarrow$ **MAXIMUM POWER TRANSFER THEOREM**

To get the maximum external power from a power source with internal resistance, the resistance of the load must equal the resistance of the source.



In DC circuits,  $Rs = R_L$ 

In AC circuits, where Rs = R + jX and Load  $Z_L = R_L + jX_L$ ,

For maximum power transfer,  $Z_L = Zs^*$  (Conjugate) = Rs - jXs

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### **NETWORK THEORY**

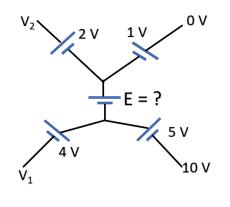
## BASICS - DC ANALYSIS AND NETWORK THEOREMS

**WORK BOOK QUESTIONS** 

#### **WORKBOOK QUESTIONS**

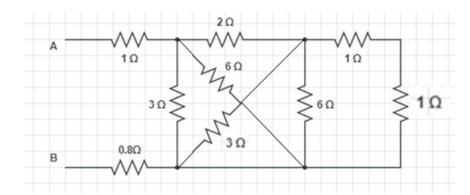
#### **TOPIC 1** $\rightarrow$ **BASIC TERMS**

Q1. In the circuit of figure the value of the voltage source E is

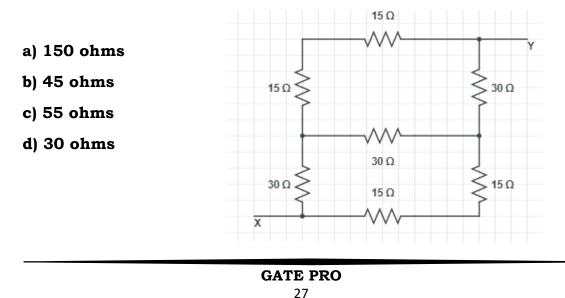


a) -16 V	b) 4 V	c) -6 V	d) 16 V
aj -10 v	• • • • • • • • • • • • • • • • • • •	€)=0 ¥	u) 10 v

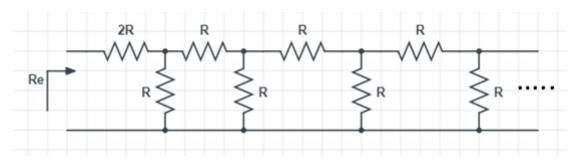
#### Q2. The equivalent resistance between the terminals A and B is $_\Omega$



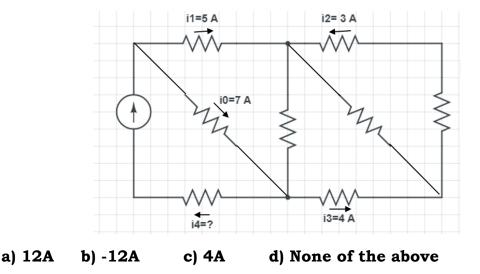
Q3. The equivalent resistance between the terminal points X and Y of the circuit shown is



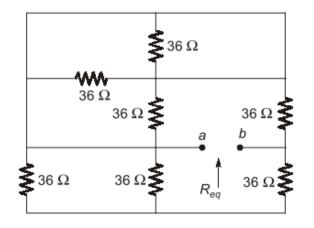
### Q4. The equivalent resistance in the infinite ladder network shown in the figure is $R_{eq} = \dots$ .



#### Q5. The current $i_4$ in the circuit of figure is equal to

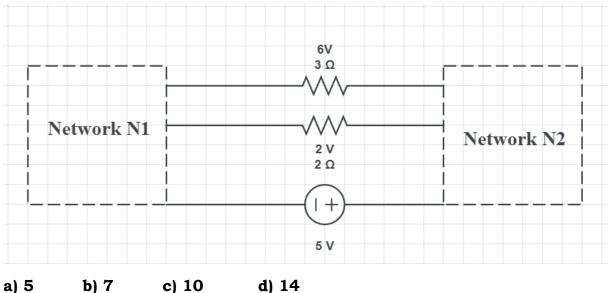


Q6. The Req between the two points is .....

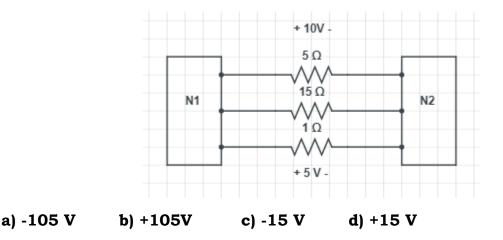


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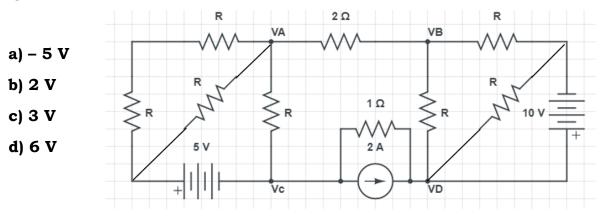
Q7. The voltages developed across the  $3\Omega$  and  $2\Omega$  resistors shown in the figure are 6 V and 2 V respectively, with the polarity as marked. What is the power (in watt) delivered by the 5 V voltage source?



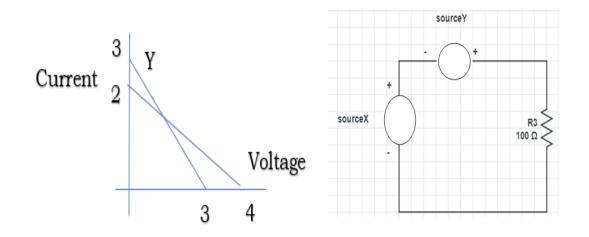
Q8. The two electrical sub networks  $N_1$  and  $N_2$  are connected through resistors as shown in the figure. Voltages across  $5\Omega$ resistor and  $1\Omega$  resistor are given to be 10 V and 5 V respectively, then voltage across  $15\Omega$  resistor is



Q9. If  $V_A - V_B = 6V$ , then  $V_C - V_D$  is



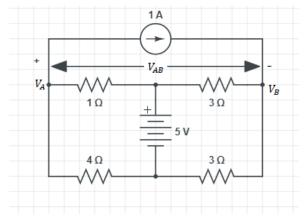
Q10. The linear I-V characteristics of 2- terminal non – ideal dc sources X and Y are shown in the figure if the sources are connected to a  $1\Omega$  resistor as shown, the current through the resistor is amperes is \_\_\_\_\_ A.



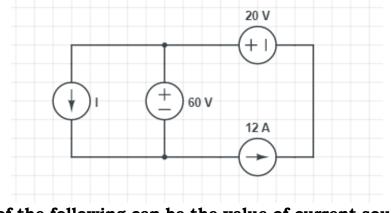
#### Q11. The potential difference $V_{AB}$ in the circuit

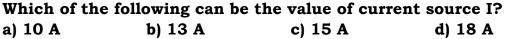
a)	0.81V	
b)	-0.8V	
C)	1.8V	

d) -1.8V

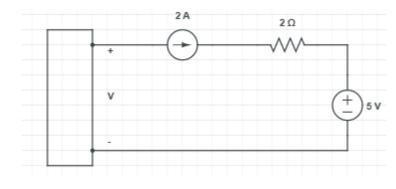


Q12. In the interconnection of ideal sources shown in the figure, it is known that the 60V source is absorbing power.



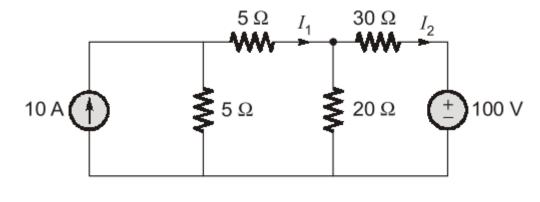


Q13. The voltage V in figure is always equal to



a) 9 V b) 5 V c) 1 V d) None of the above

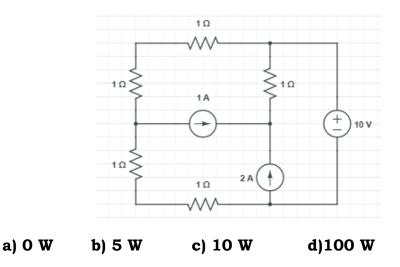
Q14. The currents  $I_1$  and  $I_2$  in the below circuit are respectively



a) 1.818 A; - 0.4545 A b) 2.451 A; - 1.568 A

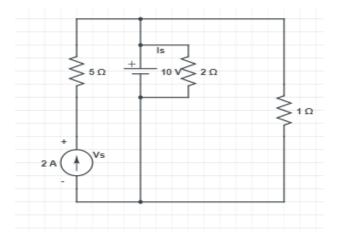
c) 0.4545A; - 1.818 A d) 1.56 A; - 2.45 A

Q15. In the circuit shown, the power supplied by the voltage source is



Q16. The current I in Amps in the voltage source, and voltage  $V_s$  in volts across the current source respectively are

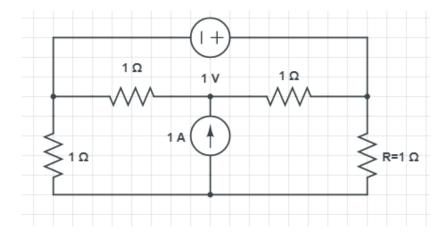
a) 13,-20 b) 8,-10 c) -8,20 d) -13,20



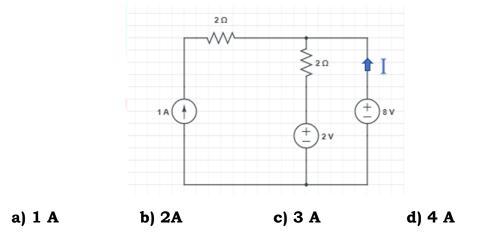
Q17. The current in the  $1\Omega$  resistor in the above problem is

a) 2A b) 3.33A c) 10A d) 12A

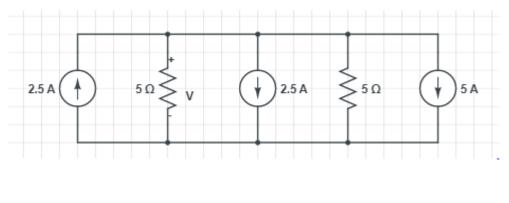
Q18. The current in amperes through the resistor R in the circuit shown in the figure is......Amps



Q19. In the circuit shown below what is the value of current I?

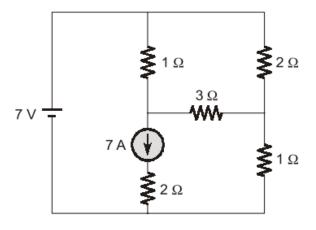


Q20. What is the voltage V in the circuit diagram?

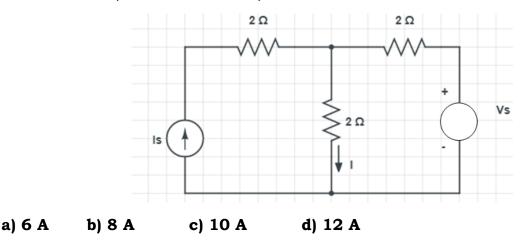


a) 7.5 V b) 16.5 V c) 12.5 V d) 14.4 V

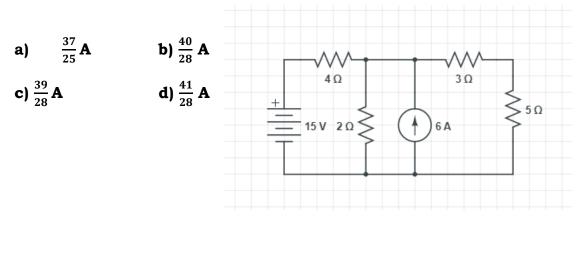
Q21. The current in the  $3\Omega$  resistance is ....



Q22. For the circuit shown below, the value of  $V_s$  is 0 when I = 4A. The value of I, when  $V_s = 16$  V, is .....

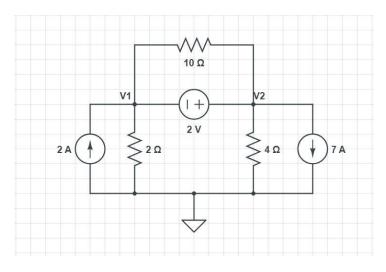


Q23. For the network shown in the figure, the current flowing the  $5\Omega$  resistance will be

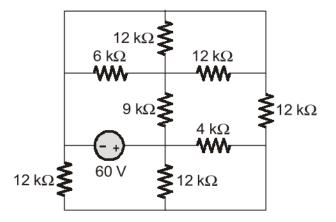


#### SURESH VSR

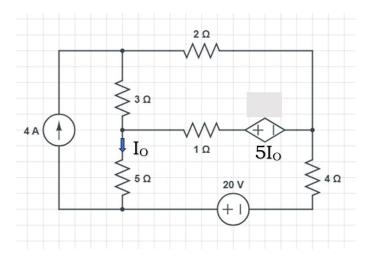
#### Q24. Power delivered by the 7A current source is



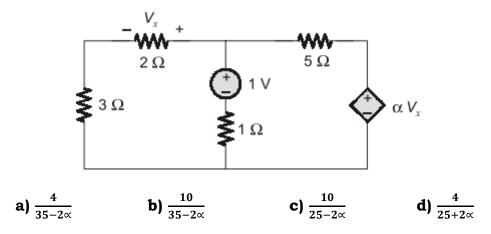
#### Q25. The power delivered to the 4K resistor is ?



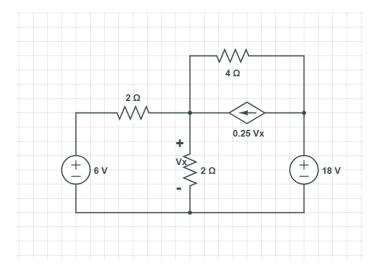
Q26. The current Io in the circuit is ....



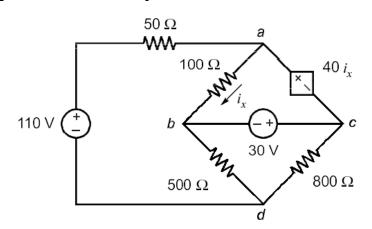
# Q27. In the following circuit voltage $V_x$ is given by



# Q28. The value of Vx in the circuit is...

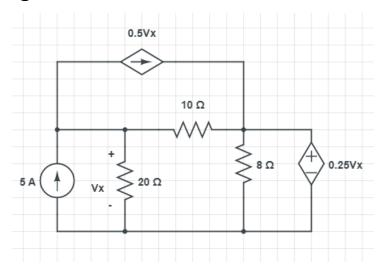


Q29. The power delivered by the 30V source is..

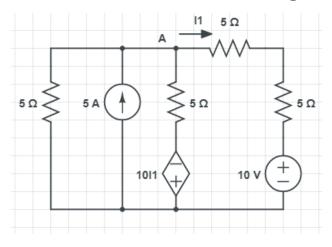


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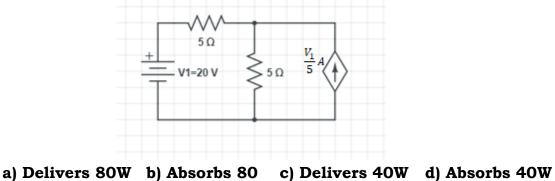
Q30. In the circuit shown, the node voltage  $V_x$  (in volts) is\_\_\_\_ Given the voltage of other node is 0.25V



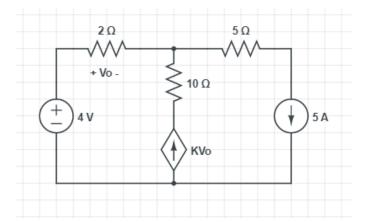
Q31. In the circuit shown below the node voltage  $V_A$  is \_\_\_\_\_V.



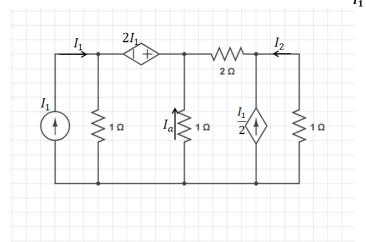
Q32. The dependent current source shown in figure



Q33 . In the given circuit, the parameter k is positive, and the power dissipated in the  $2\Omega$  resistor is 12.5W. the value of k is\_\_\_\_\_

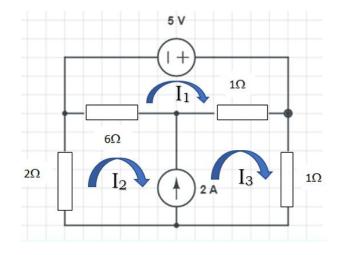


Q34. In the circuit shown below: The ratio of current  $\frac{I_2}{I_1}$  is .....

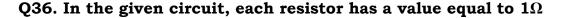


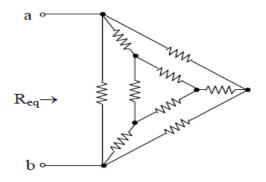
Q35. In the given circuit, the mesh currents  $I_1$ ,  $I_2$  and  $I_3$ 

- a)  $I_1 = 1A$ ,  $I_2 = 2A$  and  $I_3 = 3A$
- **b)**  $I_1 = 2A$ ,  $I_2 = 3A$  and  $I_3 = 4A$
- c)  $I_1 = 3A$ ,  $I_2 = 4A$  and  $I_3 = 5A$
- d)  $I_1 = 4A$ ,  $I_2 = 5A$  and  $I_3 = 6A$



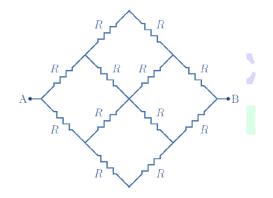
#### TOPIC 8.5 $\rightarrow$ SYMMETRY IN A NETWORK





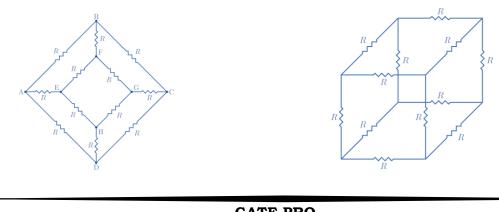
What is the equivalent resistance across the terminals a and b? a)  $\frac{1}{6} \Omega$  b)  $\frac{1}{3} \Omega$  c)  $\frac{9}{20} \Omega$  d)  $\frac{8}{15} \Omega$ 

#### Q37. Find the equivalent resistance between A and B

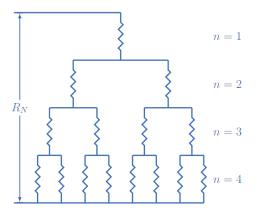


Q38. Find the equivalent resistance across

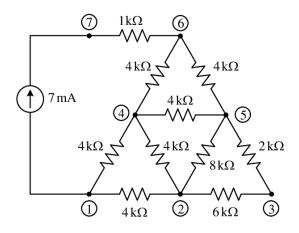
- 1) A and G = Body diagnol of a cube
- 2) A and C = Face diagnol of a cube
- 3) A and D = Adjacent vertices of a face of cube



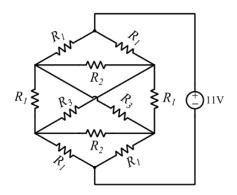
## Q39. Find the equivalent resistance Rn when n is very large



#### Q40. Find the current in each branch of the Network



Q41. In the network shown with  $R_1=1\Omega$ ,  $R_2=2 \Omega$  and  $R_3=3 \Omega$ . The network is connected to a constant voltage source of 11V.



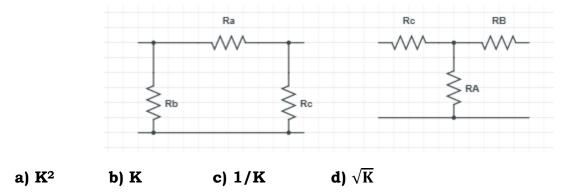
The magnitude of the current (in ampere) through the source is\_\_\_\_

#### TOPIC 8.2 $\rightarrow$ STAR - DELTA TRANSFORMATION

Q42. If each branch of a Delta circuit has impedance  $\sqrt{3}$  Z, then each branch of the equivalent Wye circuit has impedance

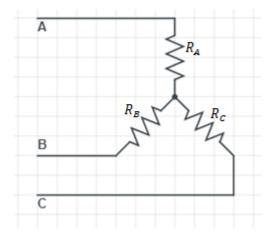
a)  $\frac{Z}{\sqrt{3}}$  b) 3Z c)  $3\sqrt{3}Z$  d)  $\frac{Z}{3}$ 

Q43. Consider a delta connection of resistors and its equivalent star connection as shown below. If all elements of the delta connection are scaled by a factor K, K>0, the elements of the corresponding star equivalent will be scaled by a factor of



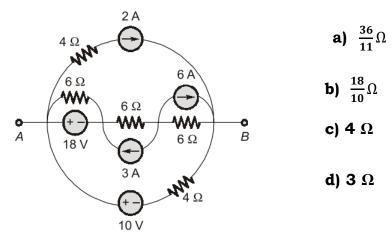
Q44. Consider the star network shown in figure. The resistance between terminals A and B with C open is  $6\Omega$ , between terminals B and C with A open is  $11\Omega$ , and between terminals C and A with B open is  $9\Omega$ . Then

**a)**  $R_A = 4\Omega, R_B = 2\Omega, R_C = 5\Omega$  **b)**  $R_A = 2\Omega, R_B = 1\Omega, R_C = 10\Omega$  **c)**  $R_A = 3\Omega, R_B = 3\Omega, R_C = 4\Omega$ **d)**  $R_A = 5\Omega, R_B = 1\Omega, R_C = 10\Omega$ 

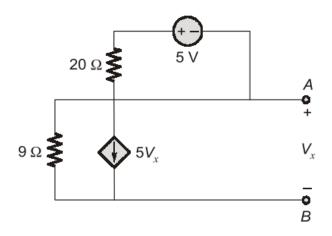


# **TOPIC 9** $\rightarrow$ **NETWORK THEOREMS**

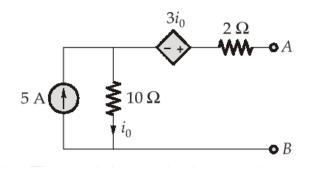
Q45. The circuit shown below, the Norton equivalent resistance across terminal AB is



Q46. The Thevenin's equivalent resistance seen across the<br/>terminal A and B of the circuit shown in the figure below isa)  $20 \Omega$ b)  $9 \Omega$ c) $6.206 \Omega$ d)  $195.65 m\Omega$ 

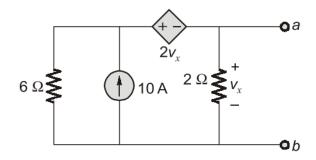


Q47.Consider the circuit shown in the figure below:

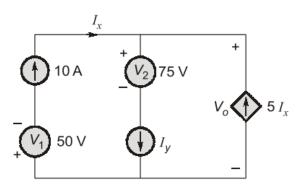


**GATE PRO** 42 The venin's equivalent resistance seen across terminals A and B is a)  $2 \Omega$  b)  $10 \Omega$  c)  $12 \Omega$  d)  $15 \Omega$ 

Q48. A load resistance  $R_L$  is to be connected between a, b such that power transferred to the load  $R_L$  is maximum. The value of  $R_L$  is \_\_\_\_\_\_\_\_.

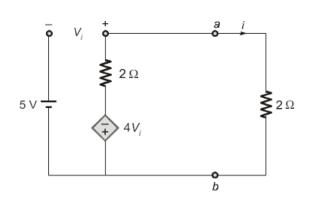


Q49. The total power developed in the circuit, if  $V_0 = 125$  V is



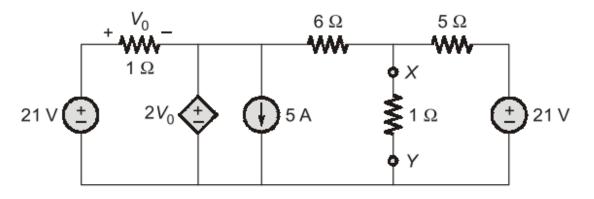
Q50. For the circuit shown in figure, Thevenin resistance is ....

a)  $2.5 \Omega$  b)  $4 \Omega$  C)  $0.4 \Omega$  d)  $0.8 \Omega$ 



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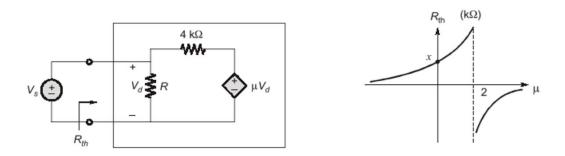
Q51. In the circuit show below,



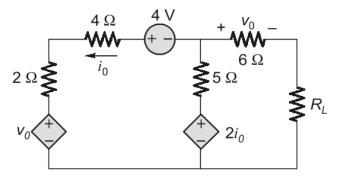
The current through 1  $\Omega$  resistance in between X Y is \_\_\_\_A.

Q52. Consider the circuit shown in Figures.

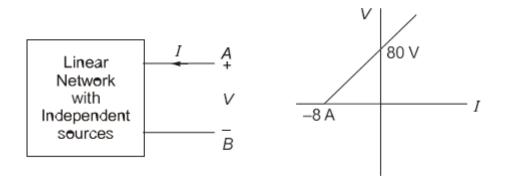
Figure A represents the variation of the Thevenin's resistance and the value of  $\mu$ . If the value of the intercept on the R<sub>th</sub> axis is equal to x then the value of x is equal to \_\_\_\_\_ k  $\Omega$ .



Q53. in the circuit shown below if maximum power is transferred to the  $R_L$ , then  $R_L$  is \_\_\_\_\_  $\Omega$ 

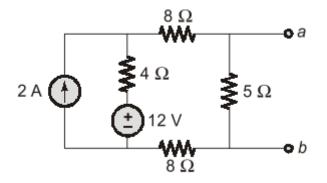


# Q54. consider a linear network with passive elements and independent source with characteristics

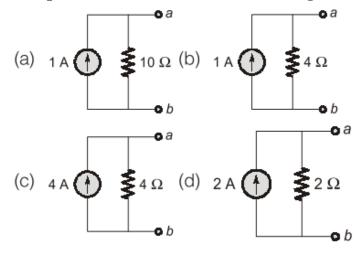


A variable resistor  $R_L$  is connected between the terminal AB, the maximum power transfer to the load  $R_L$  is \_\_\_\_\_ W.

#### Q55. Consider the circuit shown in the figure below:



The Norton equivalent circuit of the above figure can be given as,

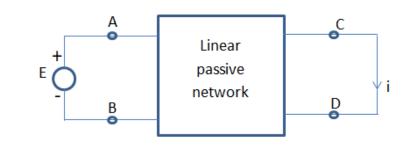


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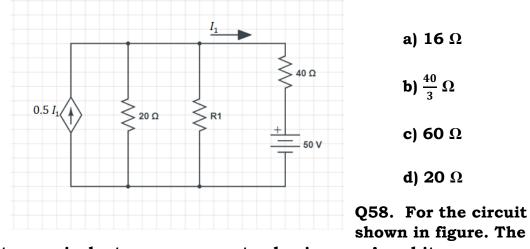
Q56. For the circuit shown in the given figure, when the voltage E is 10 V, the current i is 1 A.

If the applied voltage across terminal C-D is 100 V, the short circuit current flowing through the terminal A-B will be

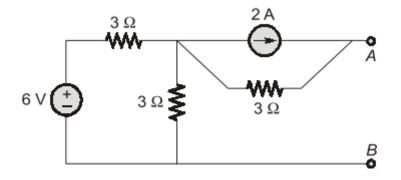




Q57. In the network of the figure, the maximum power is delivered to  $R_1$  if its value is



Norton equivalent source current value is \_\_\_\_\_ A and its resistance is \_\_\_\_\_ Ohms.



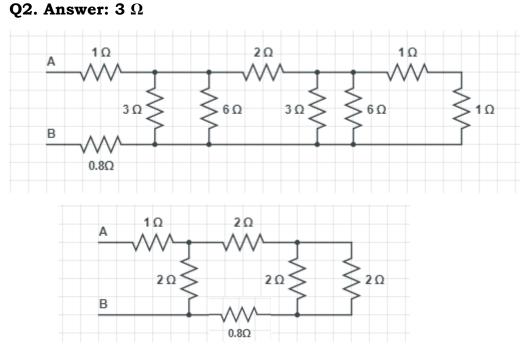
# BASICS - DC ANALYSIS AND NETWORK THEOREMS

HINTS AND KEY – WORKBOOK

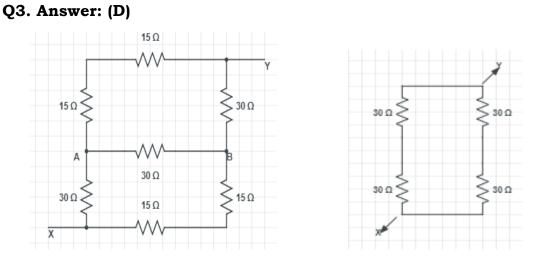
# Hints and Key WORKBOOK – QUESTIONS

# **TOPIC 1** $\rightarrow$ **BASIC TERMS**

Q1. Answer: (a) 10+5+E+1=0, E = -16V



 $R_{AB} = (2/2 + 2) / 2 + (1+0.8) = 3 \Omega$ 



 $R_{xy} = (30 + 30) // (30 + 30) = 30\Omega$ 

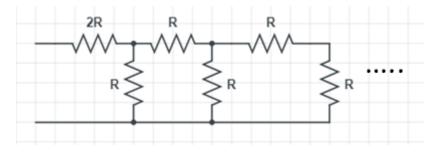
The bridge is balanced, so that eliminate AB branch.

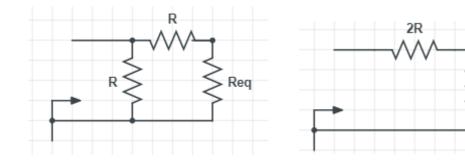
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Req

#### Q4. Answer: 2.618





 $R_{eq} = \left(\frac{-1+\sqrt{5}}{2}\right) \mathbf{R} \quad \text{and} \quad R_{eq} = \mathbf{2R} + \left(\frac{-1+\sqrt{5}}{2}\right) \mathbf{R} \quad \frac{R_{eq}}{R} = \mathbf{2.618}$ 

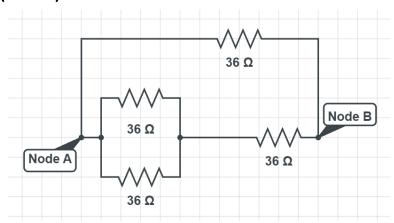
Q5. Answer: b

**Apply KCL** 

$$i_5 + i_0 + i_4 = 0$$
  $i_4 = -12A$ 

2 = 1I

**Q6.** Answer :  $(21.6\Omega)$ 



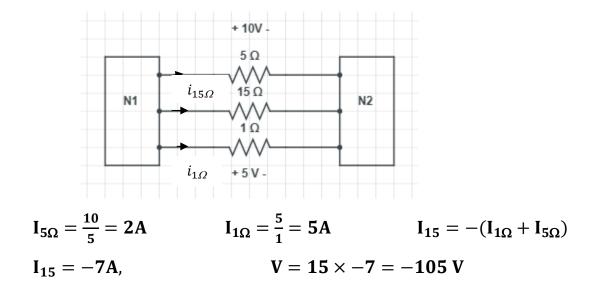
Q7. Answer: (a)

Apply KCL

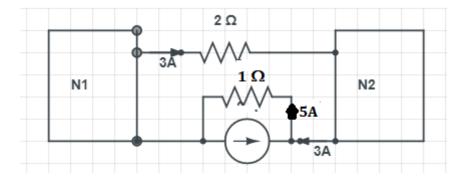
$$I = 1A$$
,  $P = 5 \times 1 = 5W$ 

Q8. Answer: (a)

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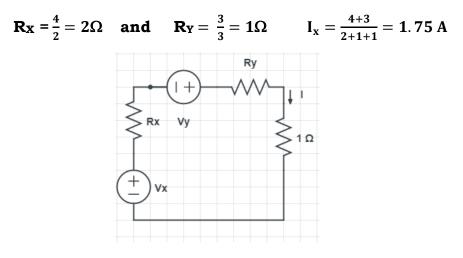
Q9. Answer: (a)



 $V_D - V_C = 5V$ 

 $\mathbf{V}_{\mathrm{C}} - \mathbf{V}_{\mathrm{D}} = -5\mathbf{V}$ 

Q10. Answer: 1.75 Ampere



**GATE PRO** 50 Q11. Answer: (b)

Apply KCL at each node

$$\frac{V_A - 5}{1} + \frac{V_A}{4} + 1 = 0; \qquad V_A = \frac{16}{5}V$$
$$\frac{V_B - 5}{1} + \frac{V_B}{3} = 1; \qquad V_B = 4V$$
$$V_A - V_B = -0.8V$$

Q12. Answer: (a)

If the current is less than 12A then only 60V source is absorbing the power From all the options I = 10A

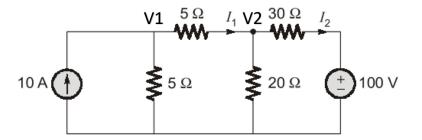
Q13. Answer: (d)

The voltage across 2 A Source is unknown so that it is not possible to find 'V'

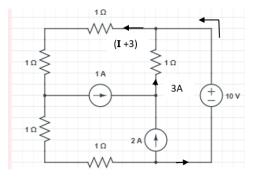
Q14. Answer: (C)

Apply Nodal analysis at V1 and V2,

V1 = 47.72V and V2 = 45.45 V



Q15. Answer: (a) Write KVL



10- (I+3) - (I+3) - (I+2) - (I+2) = 0 I = 0,  $P = V I = 10 \times 0=0w$ 

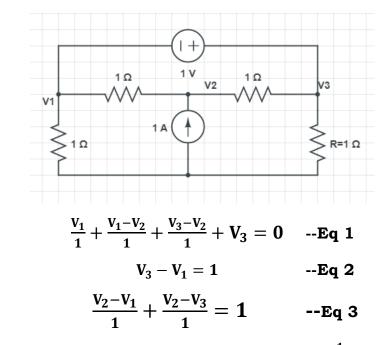
Q16. Answer: (d)

Apply KVL in first loop  $V_s -2 \times 5 - 10 = 0$   $V_s = 20V$ Current flowing through  $1\Omega$  is  $\frac{10}{1} = 10A$ Current flowing through  $2\Omega$  is  $\frac{10}{2} = 5A$ Apply KCL at node :  $2 = I_s + 10 + 5$   $I_s = -13$  Amps

Q17. Answer: (c)

$$\frac{10}{1} = 10A$$

Q18. Answer: 1A



Solve Eq 1 and Eq 2  $V_1 = 0; V_2 = 1; V_3 = 1; I_R = \frac{1}{1} = 1A$ 

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Q19. Answer: (b)

Apply KCL for current in 2V source,

 $I + 1 = \frac{8-2}{2}$  I = 2A

Q20. Answer: (c)

The current 5A divides between the two 50hm resistors.

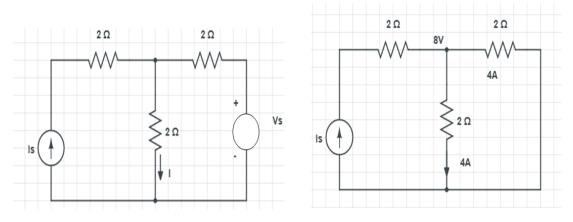
The voltage is  $2.5 \times 5 = 12.5 \text{ V}$ 

Q21. Answer: (0.5 A)

Apply nodal analysis at the unknown voltages,

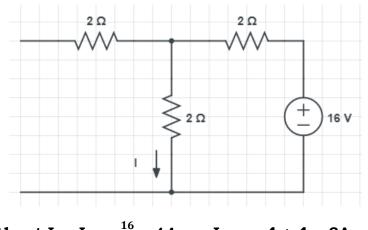
Or apply superposition theorem.

Q22. Answer: (b)



Without Vs,  $I_{2\Omega} = \frac{8}{2} = 4A$ 

 $I_s = 4 + 4 = 8A$ 



Without Is  $I_{2\Omega} = \frac{16}{4} = 4A$   $I_{total} = 4 + 4 = 8A$ 

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Q23. Answer: (c)

Write KCL equation

**6** =  $\frac{v}{2}$  +  $\frac{v-15}{4}$  +  $\frac{v}{8}$ **V** =  $\frac{78}{7}$  Volts,  $I_{5\Omega} = \frac{78}{7(5+3)} = \frac{39}{28}$  Amp.

Q24. Answer: 37.31 Watts

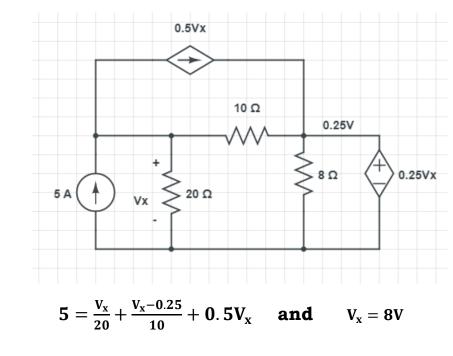
Q25. Answer: 100 W

Q26. Answer: -0.47A

Q27. Answer: B

Q28. Answer: 7.5 V

Q29. Answer: 12 Watts



Q30. Answer: 8V

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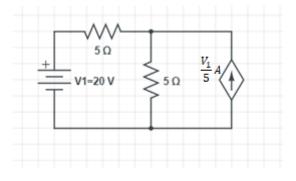
**NETWORK THEORY** 

#### SURESH VSR

#### Q31. Answer: 11.42 V

$$\frac{V_A}{5} + \frac{V_A + 10I_1}{5} + \frac{V_A - 10}{10} = 5$$
$$I_1 = \frac{V_A - 10}{10} \qquad V_A = 11.42 V$$

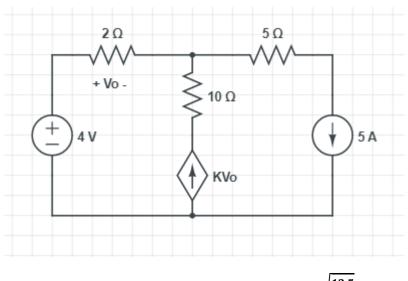
#### Q32. Answer: (a)



 $\frac{\frac{V_1}{5}}{\frac{20}{5}} = \frac{20}{5} = 4A$  $\frac{\frac{V-20}{5}}{\frac{1}{5}} + \frac{\frac{V}{5}}{\frac{1}{5}} = 4; \qquad \mathbf{V} = 20\mathbf{V}$ 

Power = **20×4 = 80W** (delivers 80W)

# Q33. Answer: 0.5



**Power in Resistor** = 12.5,

$$I = \sqrt{\frac{12.5}{2}} = 2.5A$$

#### Applying KCL at node

 $2.5+kV_0 = 5$ ,  $kV_0 = 2.5$ , k(2+2.5) = 2.5; k = 0.5

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Q34. Answer: - 0.786 Watts

Q35. Answer: (a)

An ideal current source is located between 2 and 3 so it is a super loop. Apply KVL for both loops at a time

#### Write KVL for all 3 Loops

$8I_1 + 2I_3 - 7I_2 = 0$	<b>Equation</b> -1
$I_3 - I_1 = 2$	Equation -2
-6I <sub>1</sub> + 7I <sub>2</sub> + I <sub>3</sub> =5	Equation -3

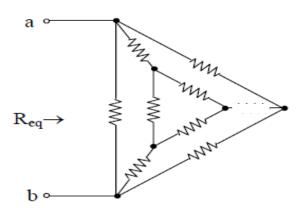
Solve 1, 2 & 3

 $I_1 = 1A, I_2 = 2A \text{ and } I_3 = 3A$ 

TOPIC 8.5  $\rightarrow$  SYMMETRY IN A NETWORK

Q36. Answer: (D)

The problem involves mirror symmetry and once resistance can be removed as it has no current flowing.



Req = 1 / / 2 / / (2 + 2/3) = 8/15

Q37. Answer: 3R/2

The problem involves folding symmetry and resistors are  $\frac{1}{2}$  R

Q38. Answer:

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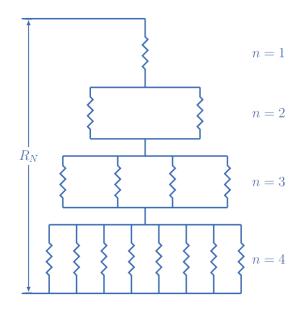
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1) A and G = 5R/6

2) A and C = 3R/4

3) A and D = 7R/12

Q39. Answer: 2R



The equipotential points can be joined as shown, Req of n<sup>th</sup> level = R /  $2^{(n-1)}$ , R final of the network = R ( 1 + 1/2 + 1/4 + 1/8 + .....1/( $2^{(n-1)}$ ) This is geometric progression with R final = 2R(1 - 1/2<sup>n</sup>) When n is large enough, R final = 2R

Q40. Answer: Current from 6 to 4 to 1 = 4mACurrent from 6 to 5 = 3mACurrent from 5 to 4 to 2 = 1mACurrent from 5 to 3 to 2 = 1mACurrent from 5 to 2 = 1mACurrent from 5 to 2 = 1mA

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**NETWORK THEORY** 

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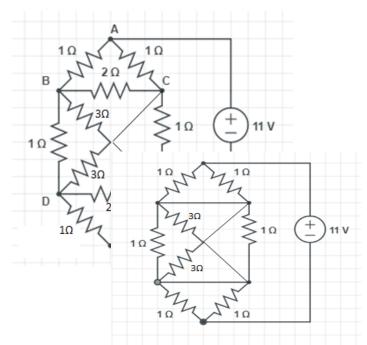
Q41. Answer: (a)

As per the symmetry of the network,

The node voltages at B & C same and D & E are same.

$$R_{T} = (1 ||1) + (1||3) + (1||3) + (1||3) + (1||1) = \frac{11}{8}\Omega,$$

$$I = \frac{11}{11/8} = 8A$$



#### TOPIC 8.2 $\rightarrow$ STAR - DELTA TRANSFORMATION

Q42. Answer: (a)

$$Z_{Y} = \frac{\sqrt{3}Z}{3} = \frac{Z}{\sqrt{3}}$$

Q43. Answer: (b)

$$R_{c} = \frac{R_{a} \cdot R_{b}}{R_{a} + R_{b} + R_{c}}$$

$$R_{c} = \frac{kR_{a} \cdot kR_{b}}{kR_{a} + kR_{b} + kR_{c}} = K \cdot R_{c}$$

$$R'_{b} = KR_{b} \qquad R'_{a} = KR_{a}$$

Q44. Answer: (b)

**A+B** = **6**Ω; **B+C** = **11**Ω; **A+C** = **9**Ω

From the options: verify it

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## **TOPIC 9** $\rightarrow$ **NETWORK THEOREMS**

- Q45. Answer: (D)
- Q46. Answer: (D)
- Q47. Answer: (D)
- Q48. Answer: 1 Ohm
- Q49. Answer: 8000 Watts
- Q50. Answer: Answer (C)
- Q51. Answer: 4.78 A
- Q52. Answer: 2K
- Q53. Answer: 7.33 Ohms
- Q54. Answer: 160 Watts
- Q55. Answer: (B)
- Q56. Answer: (C)
- Q57. Answer: (A)
- Q58. Answer: 4.50hms

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**NETWORK THEORY** 

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# **STEADY STATE AC ANALYSIS**

**THEORY – SHORT NOTES** 

# CHAPTER 2 AC ANALYSIS AND RESONANCE

**TOPIC 1**  $\rightarrow$  Steady State AC analysis DC voltage stands for Direct current voltage

DC voltage has a constant value at any time.

DC current has unidirectional flow of electrons at a constant velocity.

AC voltage stands for Alternating current voltage

The voltage or current changes it's polarity and hence the direction of moving electrons periodically with time

A typical AC voltage is harmonic in nature whose mathematical function is

A sin( $\theta$ ) or A cos( $\theta$ ) or A  $e^{j\theta}$ 

A = Amplitude of the waveform

 $\theta$  = Phase of the waveform

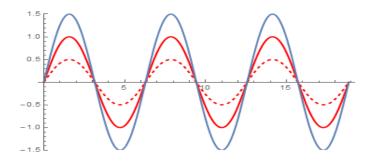
Phase  $\theta$  is a linear function of time for simple harmonics

 $\theta = \omega \mathbf{t} + \phi$ 

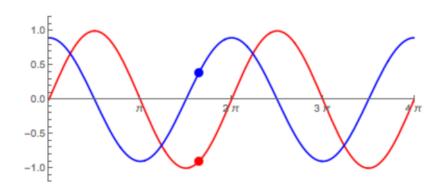
 $\omega = 2 \pi f = angular frequency (Radian / seconds)$ 

f = Frequency of the Harmonic (Hertz)

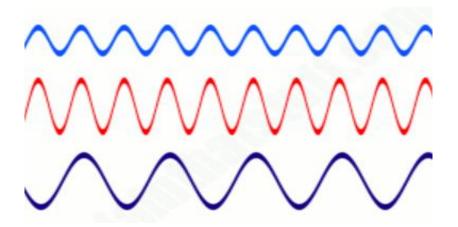
 $\phi$  = Initial phase of the waveform or Phase delay



Harmonics of same initial phase and frequency



Harmonics of same amplitude & frequency with phase delay



Harmonics of different frequencies , amplitudes and phase delay

# **TOPIC 2** $\rightarrow$ Complex numbers and their importance

Every DC voltage or current is a scalar with a magnitude which is equal to the value of the V or I

Every AC voltage or current is a phasor or vector with a Magnitude equal to it's amplitude and Direction being the phase  $\theta$ Phasor notations  $A \angle \theta$  represents a harmonic

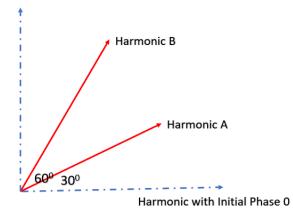
In circuits where frequency remains to be constant in all elements, the voltages and currents might have a phase delay between themselves,

The phase  $\theta$  is replaced by phase delay  $\phi$ 

Example :

A $\angle$ 30 and B $\angle$ 70 are two harmonics of same frequency delayed by 40° different amplitudes

The phasor diagram of the above example is shown below.



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# **TOPIC 3** $\rightarrow$ Reactance and its importance

Reactance is frequency dependent resistance exhibited by Inductors and Capacitors.

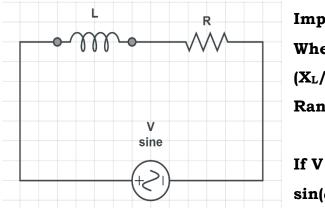
It is represented as  $X_L$  or  $X_C$ ,  $X_L = \omega L$  and  $X_C = \frac{1}{\omega C}$ In an Inductor,  $V = L \frac{dI}{dt}$  when  $I = I_0 \sin(\omega t)$   $\frac{dI}{dt} = I_0 \omega \cos(\omega t) = j \omega I_0 \sin(\omega t)$  ( $j = e^{j90}$ )  $V = j\omega L I = I jX_L \rightarrow Linear$  element obeying Ohm's Law The time delay of 90° is represented by j The voltage leads the current in phase by 90° In a Capacitor,  $I = C \frac{dV}{dt}$  when  $V = V_0 \sin(\omega t)$   $\frac{dV}{dt} = V_0 \omega \cos(\omega t) = j \omega V_0 \sin(\omega t)$  ( $j = e^{j90}$ )  $V = I \frac{1}{j\omega C} = I \frac{-j}{\omega C} = I \frac{1}{j} X_C = -I j X_C \rightarrow Linear$  element obeying

Ohm's Law

The voltage lags behind the current in phase by 90°

# **TOPIC 4** $\rightarrow$ Simple Impedance circuits

### TOPIC 4.1 $\rightarrow$ Series R-L circuit

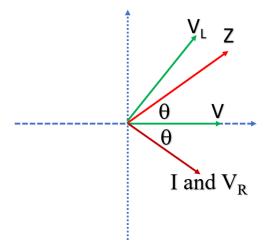


Impedance  $Z = R + jX_L = |Z| \angle \theta$ , Where  $X_L = \omega L$  and  $\theta = \tan^{-1}$ ( $X_L/R$ ) Range of phase  $\theta$  is from [0 - 90]

If  $V = V_m \sin(\omega t)$  then  $I = I_m \sin(\omega t - \theta)$ 

Voltage and Current are shifted in phase by  $\boldsymbol{\theta}$ 

Voltage across the resistance is in phase to current Voltage across the inductance is 90° shifted to current.



TOPIC 4.2  $\rightarrow$  Series R-C circuit

All the aspects same as RL circuit, except  $\theta$ Impedance Z = R - jX<sub>C</sub> =  $|Z| \angle \theta$ X<sub>C</sub> =  $\frac{1}{\omega C}$  and  $\theta$  = tan<sup>-1</sup> (- X<sub>C</sub>/R) Range of phase  $\theta$  is from [90 - 180] TOPIC 4.3  $\rightarrow$  Series R-L-C circuit

All the aspects same as RL or RC circuit, except  $\theta$ Impedance Z = R + j(X<sub>L</sub> - X<sub>C</sub>) = R + jX =  $|Z| \ge \theta$ ,  $\theta$  = tan<sup>-1</sup>(X/R) Range of phase  $\theta$  is from [0 - 180] and depends on X<sub>L</sub> > X<sub>C</sub> or X<sub>L</sub> < X<sub>C</sub>

### **TOPIC 5** $\rightarrow$ **Power Calculations in Impedance circuits**

Only resistance can dissipate a finite non-zero average power, As V and I are in phase, Average Power dissipated in a  $R = V_{RMS} X I_{RMS}$ 

Total power dissipated in a Reactive element =  $\int_0^T V(t) I(t) dt = 0$ 

As V(t) and I(t) are out of phase out of  $90^{\circ}$ ,

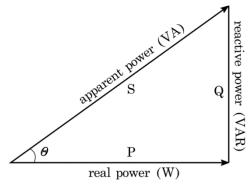
Real Power dissipated in a reactive element is zero

In general,

Real Power Dissipated I<sup>2</sup>R or V<sup>2</sup>/R = I<sup>2</sup> Z  $\cos\theta$  = V I  $\cos\theta$  (Watts) Reactive Power I<sup>2</sup>X or V<sup>2</sup>/X = I<sup>2</sup> Z  $\sin\theta$  = V I  $\sin\theta$  (VAR)

**Apparent Power = V I** 

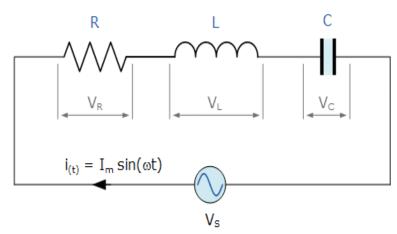
Power Factor =  $\cos\theta$ Power Triangle



# TOPIC 6 $\rightarrow$ Resonance

**TOPIC 6.1**  $\rightarrow$  Series Resonance (Acceptor circuit)

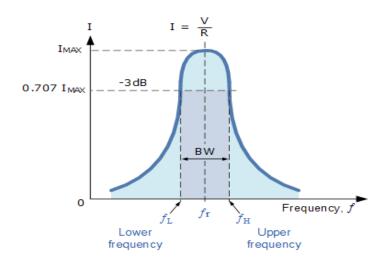
The circuit offers minimum impedance, acting as a bandpass filter at a specific frequency called as resonant frequency.



A special case in series R-L-C circuit where  $X_L = X_C$  is called as resonance

At resonance, 
$$\omega L = \frac{1}{\omega C}$$
, Resonant Frequency  $\omega_0 = \frac{1}{\sqrt{L C}}$ 

Impedance in the circuit is minimum,  $Z_{min} = R$ Current in the circuit is maximum,  $I_{max} = V/R$ 



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#### **NETWORK THEORY**

 $\omega_L$  and  $\omega_H$  are the lower and higher cut-off frequencies of the resonance curve, where the output power is  $\frac{1}{2}$  the maximum power at resonance.

The current here is  $\frac{1}{\sqrt{2}}$  times the maximum value at resonance.

The current in general is, 
$$I = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$
  $I_{max} = \frac{V}{R}$   
When  $R = \omega L - \frac{1}{\omega C}$ ,  $I = \frac{1}{\sqrt{2}} I_{max}$ ,  $\omega = \omega_L \text{ or } \omega_H$   
 $\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$   $\omega_H = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$   
Bandwidth of the resonant circuit  $BW = \omega_H - \omega_L = \frac{R}{L}$   
The geometric mean of  $\omega_H$  and  $\omega_L$  is  $\omega_o = \sqrt{\omega_L \omega_H}$   
Quality factor of the circuit  $= \frac{\text{Resonant Frequency}}{\text{Bandwidth}} = \frac{\omega L}{R} = \frac{1}{\omega CR}$ 

#### Voltages across each element versus frequency

At resonant frequency, voltage across resistor is maximum and it's value is equal to source voltage.

Capacitive Reactance dominates at low frequencies  $f < f_0$ Voltage across the capacitor is maximum at  $f_1 < f_L$  (Lower cut-off) This value is larger than source voltage (Voltage magnification)

$$\mathbf{f}_1 = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{R^2C}{2L}}$$

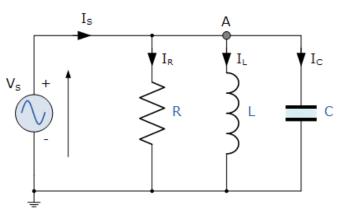
Inductive nature dominates at higher frequencies  $f > f_0$ Voltage across the inductor is maximum at  $f_2 > f_H$  (Upper cut-off) This value is larger than source voltage (Voltage magnification)

$$\mathbf{f_2} = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{R^2C}{2L}}$$

Note that fo = Resonant frequency =  $\sqrt{f_1 x f_2}$ 

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# TOPIC 6.2 $\rightarrow$ Shunt Resonance (Rejector Circuit)



At resonance,  $\omega C = \frac{1}{\omega L}$ , Resonant Frequency  $\omega_0 = \frac{1}{\sqrt{LC}}$ 

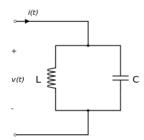
Impedance in the circuit is maximum,  $Z_{max} = R$ 

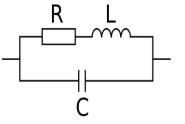
Current in the circuit is minimum,  $I_{min} = V/R$ 

Current magnification occurs similar to voltage in series resonance

Special case of shunt resonant circuit without R is called Tank circuit.

A non ideal tank circuit has series resistance along with inductor.





Tank circuit

Non ideal tank circuit

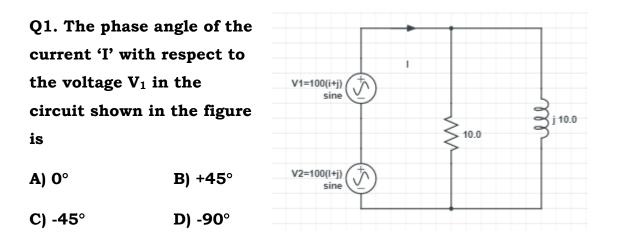
# **STEADY STATE AC ANALYSIS**

# **WORK BOOK QUESTIONS**

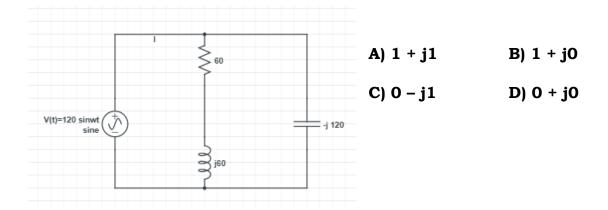
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# **WORKBOOK QUESTIONS**

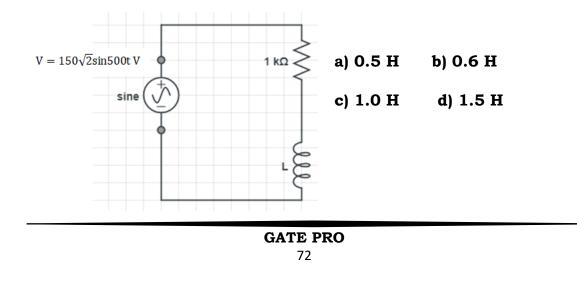
# **TOPIC 1** $\rightarrow$ Steady state AC analysis



#### Q2. For the circuit given below. What is the value of I?

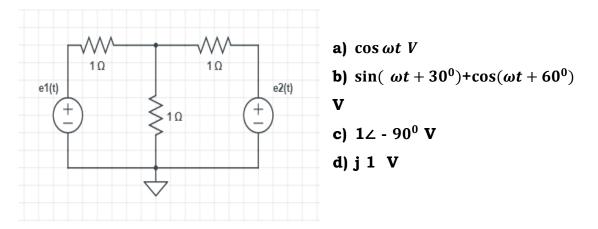


Q3. For the AC circuit as shown below, if the rms voltage across the resistor is 120 V. what is the value of the inductor?

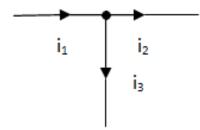


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Q4.In the circuit shown in the below figure,  $e_1(t) = \sqrt{3}\cos(\omega t + 30^0)$  and  $e_2(t) = \sqrt{3}\sin(\omega t + 60^0)$ . What is the voltage v(t) across the 1  $\Omega$  grounded resistor ?

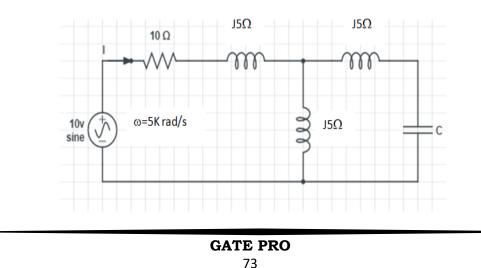


Q5. Three currents  $i_1, i_2$  and  $i_3$  meet at a node as shown in the figure below. If  $i_1 = 3 \cos(\omega t)$  ampere  $i_2 = 4 \sin(\omega t)$  ampere and  $i_3 = I_3 \cos(\omega t+\theta)$  ampere, the value of  $I_3$  in ampere is\_\_\_\_\_

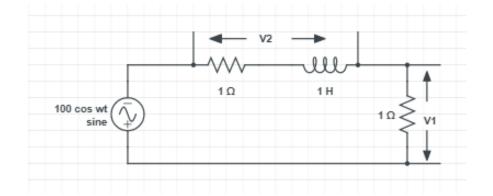


Q6. In the given circuit, the value of capacitor C that makes

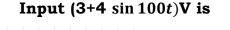
current I = 0 is  $\mu F$ 

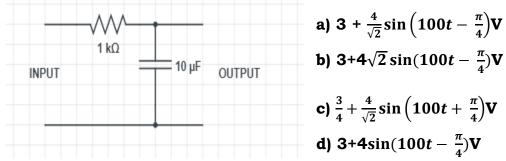


Q7. In the circuit shown the positive angular frequency  $\omega$  (in radians per second) at which the magnitude of the phase difference between the voltage V<sub>1</sub> and V<sub>2</sub> equals ......radians.

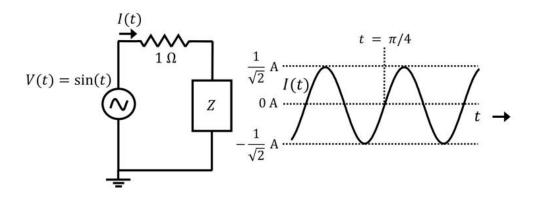


#### Q8. In figure the steady state output corresponding to the





Q9. Consider the circuit shown in the figure with input V (t) in volts. The sinusoidal steady state current I (t) flowing through the circuit is shown graphically (where t is in seconds). The circuit elements Z can be\_\_\_\_\_



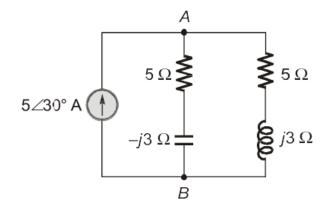
a) a capacitor of 1 F



c) an capacitor of  $\sqrt{3}$  H

d) an inductor of  $\sqrt{3}$  H

### Q10. In the AC network shown, the phasor voltage $V_{AB}$ (in volts) is



a) 0 b)  $5 \angle 30^{\circ}$  c)  $12.5 \angle 30^{\circ}$  d)  $17 \angle 30^{\circ}$ 

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+

3Ω

- j3 Ω

3Ω

j3 Ω

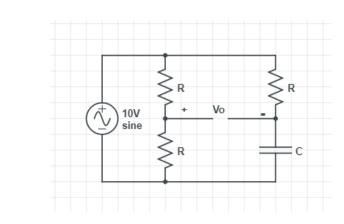
sine

30 V

Q11.In the circuit shown in the Fig. output  $\left| V_{0}(j\omega) \right|$  is

a) Indeterminable as values of R and C are not given

- b) 2.5 V
- c) 5  $\sqrt{2}$  V
- d) 5V

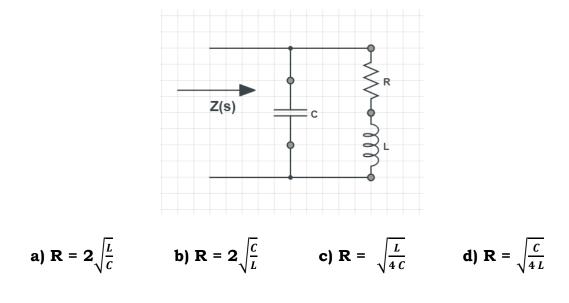


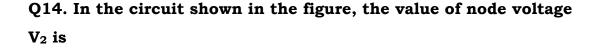
Q12. In the circuit shown in the given figure, the voltmeter indicates 30 V. The reading of the ammeter will be

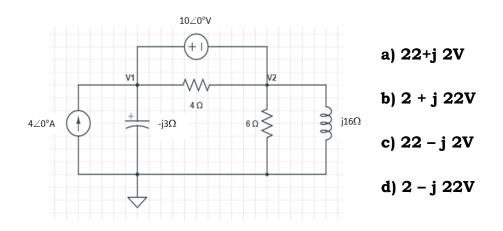
- A) 20 A B) 10  $\sqrt{2}$  A
- C) 10 A D) Zero
- Q13. The poles of the

impedance function in the circuit shown below will be real

and coincident when

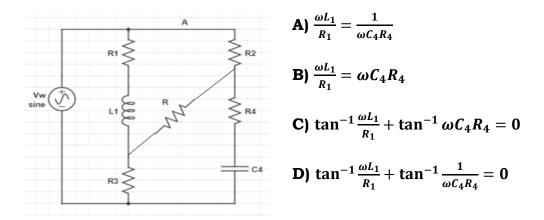






Q15. In the circuit shown in the figure,

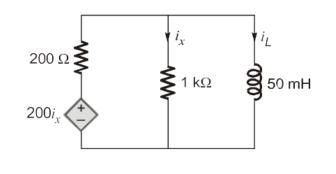
If the current in resistance 'R' is zero, then



Q16. A parallel RLC circuit has  $R = 1\Omega$  with a current source across the circuit is  $I_s = 10 \cos w_0 t$ , where  $w_0 = \frac{1}{(LC)^{1/2}}$  the current through the inductor is given by

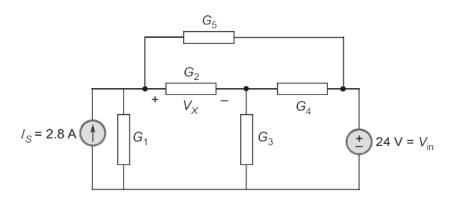
a) 0  
b) 
$$\frac{10}{w_0 L} \cos w_0 t$$
  
c)  $-\frac{10}{w_0 L} \sin w_0 t$   
d)  $\frac{10}{w_0 L} \sin w_0 t$ 

Q17. In the circuit given, the equivalent resistance across inductor is



a)  $100 \Omega$  b)  $200 \Omega$  c)  $400 \Omega$  d)  $600 \Omega$ 

Q18. consider the circuit shown in the figure below:



The value of admittance shown in the figure are equal to  $G_1 = G_2 = 2G_4 = 0.2S, G_3 = 0.3S$  and  $G_5 = 0.4S$ , then voltage  $V_x =$ 

**TOPIC 5**  $\rightarrow$  **Power Calculations in Impedance circuits** 

Q19. A voltage v(t) =  $173\sin(314t + 10^0)$  is applied to a circuit. It causes a current flow described i(t) =  $14.14 \sin(314t - 20^0)$ 

The average power delivered in nearly

a) 2500W b) 2167W c) 1060 W d) 1500W

Q20. The voltage across and the current through a load are expressed as follows V(t) =  $-170\sin\left(377t - \frac{\pi}{6}\right)V$  I(t) =  $5\cos\left(377t + \frac{\pi}{6}\right)A$  The average power in watts (round off to one decimal place) consumed by the load is\_\_\_\_\_

Q21. The average power delivered to an impendence (4- j3)  $\Omega$  by a current  $5\cos(100\pi t + 100)$ A is,

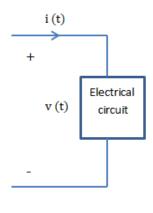
a) 44.2 W b) 50 W c) 62.W d) 125 WI

Q22. The current i(t) through a  $10\Omega$  resistor in series with an inductance is given by i(t)=3+4sin(100t+45°)+4 sin(300t + 60°)A.

The RMS value of the current and the power dissipated in the circuit are respectively

(a) 5A and 150W	(b) 11A and 250W
(c) 5 A and 250W	(d) 11A and 150W

Q23. The voltage across the circuit in the figure, and the current through it, are given by the following expressions:

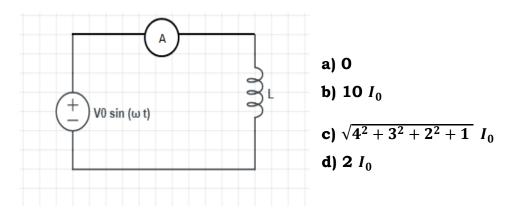


i(t)=5+Xcos(ωt) A

Where =100π radian/s If the average power delivered to the circuit is zero, then the value of X (in Ampere) is\_\_\_\_\_ (up to 2 decimal places).

Q24. When a voltage Vo sin  $(\omega_0 t)$  is applied to the pure inductor, the ammeter shown in the figure reads I<sub>0</sub>. If the voltage applied is, Vo sin  $(\omega_0 t)$  + 2 Vo sin  $(2\omega_0 t)$  + 3V<sub>0</sub> sin  $(3\omega_0 t)$ +4 Vo sin  $(4\omega_0 t)$ ,

#### the ammeter reading would be

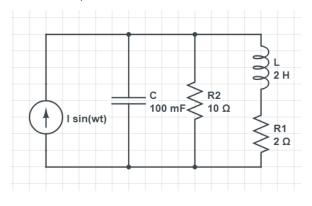




#### Q25. The resonant frequency of the circuit is .....

- a) 2 rad / second b) 20 rad / second
- c) 4 rad / second

d) 40 rad / second

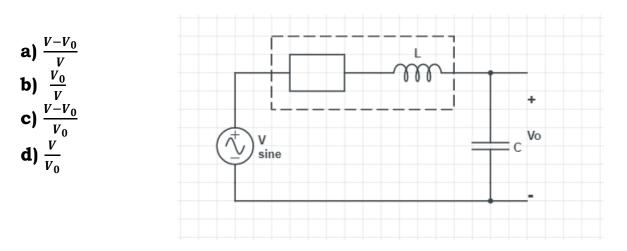


Q26. The phase difference between the Voltage and Current in a series RLC circuit is ...... when the voltage across the inductor is maximum.

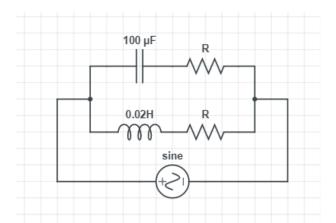
a) 45 b) 90 c) 0 d) -90

Q27. Fig. shows a circuit which has a coil of resistance R and inductance L. at resonance, the Q-factor of the coil is given by.

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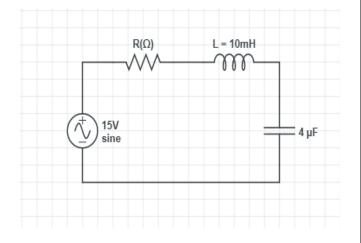


Q28. The circuit below is excited by a sinusoidal source. The value of R. in  $\Omega$ , for which the admittance of the circuit becomes a pure conductance at all frequencies is\_\_\_\_\_



Q29. A series R-L-C circuit is excited with an AC voltage source. The quality factor (Q) of the circuit is given as Q = 30.

The amplitude of current in ampere at upper half-power frequency will be\_\_\_\_\_



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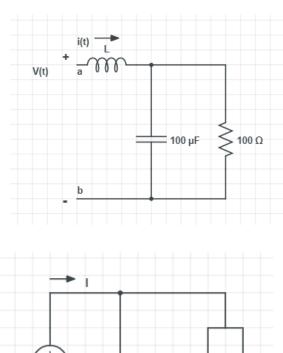
#### **NETWORK THEORY**

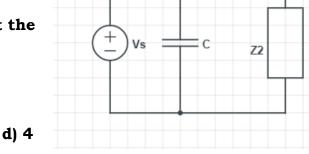
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Q30. The voltage (t) across the terminals a and b as shown in the figure, is a sinusoidal voltage having a frequency  $\omega$ = 100 radian/s. When the inductor current (t) is in phase with the voltage v(t), the magnitude of the impedance Z (in  $\Omega$ ) seen between the terminals a and b is\_\_\_\_\_ (up to 2 decimal places

Q31. In the circuit shown,  $V_s = V_m \sin 2t$  and  $Z_2=1+j$ . The value of C is chosen such that the current I is in phase with  $V_s$ . The value of C (in farads) is

(a) 
$$\frac{1}{4}$$
 b)  $\frac{1}{2\sqrt{2}}$  (c) 2



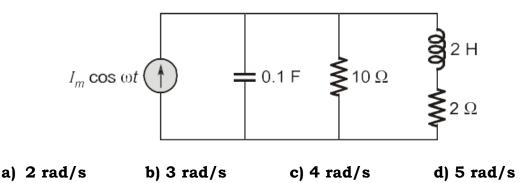


Q32. Calculate the resonant frequency of a non-ideal tank circuit. a)  $\frac{1}{2\pi\sqrt{LC}}$  b)  $\frac{1}{2\pi\sqrt{LC}}\sqrt{1-R^2\frac{C}{L}}$  (c)  $\frac{1}{2\pi\sqrt{LC}}\sqrt{1-\frac{L}{R^2C}}$  (d)  $\frac{1}{2\pi\sqrt{LC}}\sqrt{1-R^2\frac{L}{C}}$ 

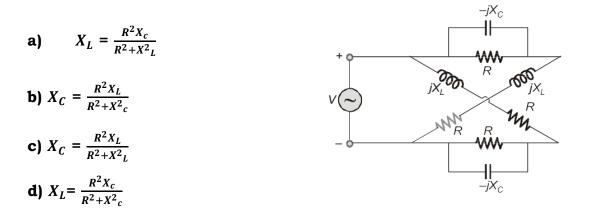
Q33. Which of the following terms correctly represents the upper and lower cut-off frequencies of a series R-L-C circuit?

(a)  $f_r \sqrt{1 + (\frac{1}{2Q})^2} + f_r \frac{1}{2Q}$  and  $f_r \sqrt{1 + (\frac{1}{2Q})^2} - f_r \frac{1}{2Q}$ (b)  $f_r \sqrt{1 + (\frac{1}{Q})^2} + f_r \frac{1}{Q}$  and  $f_r \sqrt{1 + (\frac{1}{Q})^2} - f_r \frac{1}{Q}$ (c)  $f_r \sqrt{1 + (\frac{2}{Q})^2} + f_r \frac{1}{Q}$  and  $f_r \sqrt{1 + (\frac{2}{Q})^2} - f_r \frac{1}{Q}$ (d)  $f_r \sqrt{1 - (\frac{1}{Q})^2} + f_r \frac{1}{Q}$  and  $f_r \sqrt{1 - (\frac{1}{Q})^2} - f_r \frac{1}{Q}$ 

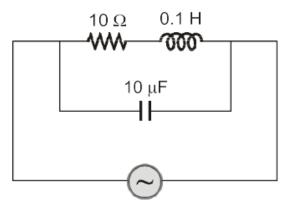
#### Q34. The following circuit shown in figure resonate



Q35. The condition on R,  $X_L$  and  $X_C$  such that current is in phase with applied voltage will be



#### Q36. For the tank circuit shown in figure, the resonant frequency



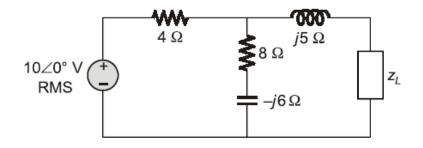
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SURESH VSR

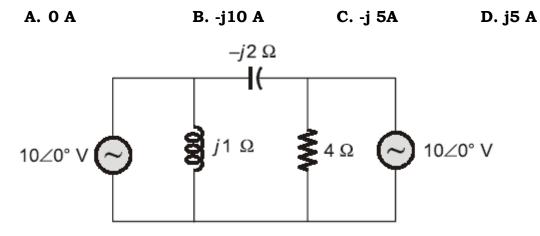
### TOPIC 7 $\rightarrow$

#### **Network Theorems in AC Analysis**

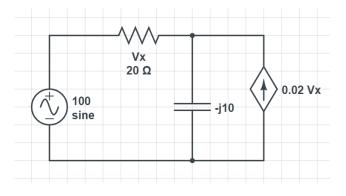
#### Q37. What is the maximum power transferred to the load ?



#### Q38. The current through the inductive reactance is ..



Q39. Find the Thevenin resistance and voltage of network shown below



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# **STEADY STATE AC ANALYSIS**

# **KEY AND HINT -WORK BOOK**

# Key & Hints WORKBOOK QUESTIONS

## **TOPIC** 1 $\rightarrow$ Steady State AC analysis

Q1. Answer: (c)

$$\mathbf{I} = \frac{200(1+j)}{10} + \frac{200(1+j)}{j10} = 20(1+j) - j20(1+j) = 40\mathbf{A}$$

$$V$$
 = 100+j100 =  $100\sqrt{2} \angle 45^\circ$  and I =  $20 \angle 0^\circ\,$  , Phase difference 45

Q2. Answer: (b)

=

$$\mathbf{I} = \frac{120}{60+j60} + \frac{120}{-j120} = \mathbf{1} + \mathbf{j0} \mathbf{A}$$

Q3. Answer: (d)

$$V_L = \sqrt{150^2 - 120^2} = 90V$$
  
 $I = \frac{120}{1k} = 120mA$ 

 $V = I \omega L = 90 \implies L = 1.5 H$ 

Q4. Answer: (a)

Consider  $\sqrt{3}\cos(\omega t + 30^{\circ}) = \sqrt{3} \angle 30^{\circ}$ , In phasor notation.

Then,  $\sqrt{3} \sin(\omega t + 60^{\circ}) = \sqrt{3} \cos(\omega t - 90 + 60^{\circ}) = \sqrt{3} \angle -30^{\circ}$ 

Applying superposition theorem,

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 = \frac{1}{3}\sqrt{3}\angle \mathbf{30^0} + \frac{1}{3}\sqrt{3}\angle \mathbf{-30^0}$$

 $=\frac{1}{3}\sqrt{3}(\cos(\omega t + 30^{\circ}) + \sin(\omega t + 30^{\circ})) + \frac{1}{3}\sqrt{3}(\cos(\omega t + 30^{\circ}) - \sin(\omega t + 30^{\circ}))$ 

= **cos**(ω**t**)

SURESH VSR

**Q**5.

$$I_{1} = I_{2} + I_{3} \text{ and } I_{3} = I_{1} - I_{2}$$
$$I_{3} = \frac{3}{\sqrt{2}} \angle 0 - \frac{4}{\sqrt{2}} \angle -90 = \frac{5}{\sqrt{2}} \angle 53.13 = 5\cos(\omega t + 53.13)$$

Q6. For I to be zero , j5 shunt (j5 series Xc) should be infinite  $Z = \infty \text{ when } Y = 0 = j 5 + j 5 - jX_c = 0$   $X_c = 10 = \frac{1}{5 \times 10^3 \times C} \qquad C = \frac{1}{5 \times 10^4} = 20 \mu F$ 

Q7. 
$$V_1 = I(1\Omega); \quad V_2 = I Z = I(1 + j\omega)$$
  
 $\theta_1 = 0 \quad ; \quad \theta_2 = \tan^{-1}(\omega)$   
 $\theta_2 - \theta_1 = \frac{\pi}{4}; \quad \omega = 1 \text{ rad/sec}$ 

Q8. Answer: (a)

$$\begin{split} V_1 &= 3V \quad \text{at } \omega = 0 \qquad X_c = \infty \quad \text{(open circuit)} \ V_{01} \stackrel{'}{=} 3V \\ V_2 &= 4 \, \sin 100t \ , \quad \omega = 100, \qquad X_c = \frac{1}{100 \times 10^{-6} \times 10} = 1000 \ \Omega \\ V_{02} &= V_2 \ \times \frac{-j1000}{1 \times 10^3 + (-j1000)} = \frac{4}{\sqrt{2}} \sin \left( 100t - \frac{\pi}{4} \right) V \\ V_0 &= 3 + \frac{4}{\sqrt{2}} \sin \left( 100t - \frac{\pi}{4} \right) Volts \end{split}$$

Q9. Answer: (b)

**V** = sint, **I** = 
$$\frac{1}{\sqrt{2}}$$
 sin(t -  $\frac{\pi}{4}$ ), Zeq =  $\sqrt{2}$   $\angle$ 45 = 1 + j

Q10. Answer: (d)

Zeq of the circuit = 3.4 Ohms, V =  $5 \angle 30 \times 3.4 = 17 \angle 30$ 

Q11. Answer :(d)  $V_0 = V_R - V_C$ 

$$V_{R} = \mathbf{10} \times \frac{R}{R+R} = 5v \qquad V_{C} = \frac{\mathbf{10} \times \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{\mathbf{10}}{j\omega RC + 1}$$
$$|V_{0}| = \left| 5 - \frac{\mathbf{10}}{j\omega RC + 1} \right| = \left| \frac{j \ 5\omega RC - 5}{j \ \omega RC + 1} \right| = \left| 5 \right| \frac{-1 + j\omega RC}{1 + j \ \omega RC} \right| = \mathbf{5V}$$

Q12 Answer: (b)  
Current through 
$$(3-j3)\Omega = \frac{30}{3} = 10A$$
  
Voltage across  $(3-j3)\Omega = 30 - j30$ ,  
Voltage across  $(3+j3)\Omega$  is  $30 - j30$ 

Current through (3-j3) =  $\frac{30 - j30}{3+j3}$  -j10A

Reading of ammeter =  $10 - j10 = 10\sqrt{2} \angle 45^{\circ}$ ,

**RMS value = 10\sqrt{2} A** 

Q13. Impedance Z =  $\frac{(R+sL) \times 1/sC}{R+sL+1/sC} = \frac{(R+sL)}{sCR+s^2LC+1}$ 

To obtain the poles of the function, equating denominator to zero,

$$\mathbf{s} = \frac{-\operatorname{RC} \pm \sqrt{(\operatorname{RC})^2 - 4 \operatorname{LC}}}{2\operatorname{LC}} \rightarrow \operatorname{RC} = 2\sqrt{\operatorname{LC}}$$

Q14. Answer: (d)

**Applying Nodal analysis** 

$$4 \angle 0 = \frac{V_1}{-j3} + \frac{V_1 - 10}{6} + \frac{V_1 - 10}{j6}$$
,  $V_1 = 12 - j22 V$ 

Q15. Answer: (a)

$$\frac{R_3}{R_1 + R_3 + j\omega L_1} = \frac{R_4 + \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4} + R_2}$$

$$R_3 R_4 - \frac{jR_3}{\omega C_4} + R_2 R_3 = R_1 R_4 + R_3 R_4 + j\omega L_1 R_4 - \frac{jR_1}{\omega C_4} - \frac{jR_3}{\omega C_4} + \frac{L_1}{C_4}$$
Equating imaginary part to zero
$$j\omega L_1 R_4 - \frac{jR_1}{\omega C_4} = \mathbf{0} \qquad \Rightarrow \qquad \frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4}$$

Q16. Answer: (B)

Voltage across Resistance = Voltage across L and C

 $V = 1 \times 10\cos(wt)$ , as the circuit has  $X_L = X_C$ 

I in the inductor = V/wL

Q17. Answer: (B)

$$\mathbf{R_{eq}} = \frac{-V}{I_L} = \frac{-I_X \times 1000}{I_L}$$

Applying Nodal analysis at  $\frac{200 I_X - I_X 1000}{200} = I_X + I_L$ 

$$\frac{-I_X}{I_L}$$
 = 1/5 and R<sub>eq</sub> = 200

Q18. Answer: (8V)

# **TOPIC 5** $\rightarrow$ **Power Calculations in Impedance circuits**

Q19. Answer: (c)

**Power** = 
$$\frac{173}{\sqrt{2}} X \frac{14.14}{\sqrt{2}} \cos 30 = 1060$$
 Watts

Q20.

$$V(t) = -170\sin\left(377t - \frac{\pi}{6}\right) = 170\cos\left(377t + \frac{\pi}{6} + \frac{\pi}{2}\right) = 70\cos\left(377t + \frac{\pi}{3}\right)$$
$$I(t) = 8\cos\left(377t + \frac{\pi}{6}\right)$$

**Power** =  $V_{rms} I_{rms} \cos\theta = \frac{170}{\sqrt{2}} \frac{8}{\sqrt{2}} \cos\frac{\pi}{6} = 588.89$  Watts

Q21. Answer: (b)

$$\mathbf{P} = \left(\frac{5}{\sqrt{2}}\right) \times \mathbf{4} = \mathbf{50} \text{ Watts}$$

Q22. Answer: (c)

$$I_{\rm rms} = \sqrt{3^2 + \frac{1}{2}[4^2 + 4^2]} = 5A$$

**Power** =  $5^2 \times 10 = 250$  Watts

 $Q23. \qquad Power = P_1 + P_2$ 

 $P_1 = 5.5 = 25$  Watts of power absorbed

Average power delivered is equal to zero

$$25 = V_{\rm rms} I_{\rm rms} \cos \theta = \frac{10}{\sqrt{2}} \times \frac{X}{\sqrt{2}} \cos 60^{\circ} \Rightarrow X = 10$$

Q24. Answer (d)  

$$V_0 \sin \omega_0 t \rightarrow I_1 = \frac{V_0}{\omega_0 L} = I_0 \text{ (rms)}$$
  
 $2V_0 \sin 2\omega_0 t \rightarrow I_2 = \frac{2V_0}{2\omega_0 L} = I_0 \text{ (rms)}$   
 $3 V_0 \sin 3\omega_0 t \rightarrow I_3 = \frac{3V_0}{3\omega_0 L} = I_0 \text{ (rms)}$   
 $4 V_0 \sin 4\omega_0 t \rightarrow I_4 = \frac{4V_0}{4\omega_0 L} = I_0 \text{ (rms)}$   
 $I_{rms} = \sqrt{I_0^2 + I_0^2 + I_0^2} + I_0^2 = 2 I_0$ 

#### **TOPIC 6** $\rightarrow$ Resonance

**Q25.** Admittance of **R** and **X**<sub>L</sub> in series branch is  $\frac{1}{2+j\omega 2}$ 

Admittance of Capacitance in shunt is 
$$\frac{j\omega}{10}$$

Total admittance =  $\frac{1}{2+j\omega 2} + \frac{j\omega}{10} = \frac{1-j\omega}{2+2\omega^2} + \frac{j\omega}{10}$ 

Equating the imaginary part to zero,

$$\frac{\omega}{2+2\,\omega^2} = \frac{j\omega}{10} \rightarrow \omega = 2$$

Q26. Voltage across the inductance is maximum at upper cut-off frequency where  $X_L = R$ ,

The circuit impedance Z = R + jR, Hence phase shift is  $45^{\circ}$ 

Q27. Q factor is voltage across the capacitor or reactance by voltage across the resistance at resonance ,

$$Q = Vo/V$$

**Q28.**  $\mathbf{Y} = \frac{1}{R+jX_L} + \frac{1}{R-jX_C} = \frac{R-jX_L}{R^2 + X_L^2} + \frac{R+jX_C}{R^2 + X_C^2}$ 

The reactive part should be zero,

$$\frac{X_{L}}{R^{2} + X_{L}^{2}} = \frac{X_{C}}{R^{2} + X_{C}^{2}} \rightarrow R^{2} = X_{C} X_{L} \rightarrow R = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.02}{100 \, \mu F}} = 14.1\Omega$$

Q29. From Q factor and L, C values , R =  $5/3 \Omega$ 

I at resonance = 15 /R = 9Amps

I at half power frequency = 15 /  $\sqrt{2}$  = 6.36 Amps

Q30. Admittance Y of C and  $100\Omega$  is = 0.01 + j $_{00}$ C = 0.01 + j0.01

$$Z = 1/Y = 50(1-j)$$
,

At resonance Z real part only exists = 50  $\Omega$ 

Q31. At resonance, Y has zero imaginary part.

**Y** = 
$$\frac{1}{1+j}$$
 + j2**C** =  $\frac{1-j}{2}$  + j2**C** , **C** =  $\frac{1}{4}$  Farads.

Q32. For a non ideal tank circuit,

**R** +j **X** = **Z**<sub>1</sub> and  $\frac{1}{j \omega C}$  = **Z**<sub>2</sub> are in parallel

 $Y_1 + Y_2 = \frac{1}{R + j \omega L} + j \omega C$ , should have imaginary part zero.

 $Y_1 + Y_2 = \frac{R - j \omega L}{R^2 + (\omega L)^2} + j \omega C$  should have imaginary part zero.

$$\frac{-\omega L}{R^2 + (\omega L)^2} + \omega C = 0$$

$$R^2 + (\omega L)^2 = \frac{L}{C}, \qquad \omega = \frac{1}{\sqrt{LC}} \sqrt{1 - R^2 \frac{C}{L}}$$

Q33. Answer (a)

Lower cut-off frequency,

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = -\frac{f_r}{2Q} + \sqrt{\left(\frac{f_r}{2Q}\right)^2 + (f_r)^2}$$
  
Bandwidth =  $\frac{R}{L} = \frac{f_r}{Q}$ 

Q34. Answer (a)

Q35. Answer (d)

Q36. Answer (158.35 Hz)

# **TOPIC 7** $\rightarrow$ Network Theorems in AC Analysis

- Q37. Answer (4.73 Watts)
- Q38. Answer (A)
- Q39. Answer:  $V_{Th} = 57.34 \angle -55$  and  $R_{Th} = (4.7 j6.7)$  Ohms

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# **TRANSIENT ANALYSIS**

# **THEORY – SHORT NOTES**

#### **TOPIC** 1 $\rightarrow$ DC Transients in R-L and R-C Circuits

The study of V and I in inductor and capacitor containing circuits just after switching is called as transient analysis.

Given a DC voltage is applied to such circuits, and switching occurring at t=0, ( Transition occurring )

**TOPIC** 1.1  $\rightarrow$  L-C behavior after switching

t→ 0<sup>-</sup> ( Circuit state just before switching - Steady state)
 Inductor acts as a short circuit
 Current is Non zero and Voltage across it is zero.
 Capacitor acts as an open circuit
 Current is zero and Voltage across it is Non zero.

2.  $t \rightarrow 0^+$  (Circuit state just after switching ) Inductor acts as a current source, whose value is value at  $t \rightarrow 0^-$ Capacitor is as a voltage source, whose value is value at  $t \rightarrow 0^-$ 

3.  $t \rightarrow \infty$  (Circuit state long after switching – Steady state) Inductor acts as a short circuit having a non zero current Capacitor acts as an open circuit having a non zero voltage

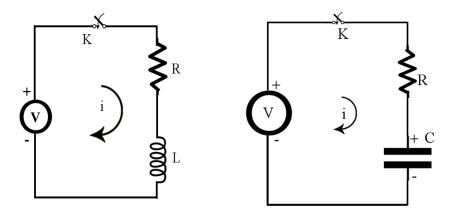
At any time in between  $0^+$  to  $\infty$ , the V or I is an exponential rise or fall depending on the switch opening or closing condition.

## **TOPIC 2** $\rightarrow$ Exponential Equations in R-L and R-C Circuits

### **TOPIC 2.1** $\rightarrow$ L-R or R-C circuit with Falling exponential

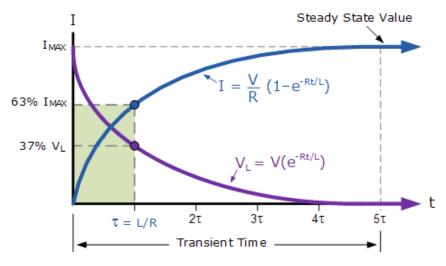
Current in the circuit I(t) =  $\frac{V}{R}e^{-\frac{t}{\tau}}$   $\tau = \frac{L}{R}$  or  $\tau = RC$ 

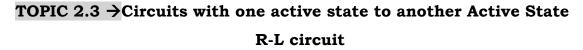
**TOPIC 2.2**  $\rightarrow$  L-R or R-C circuit with Rising exponential (Charging) (Passive to Active State)

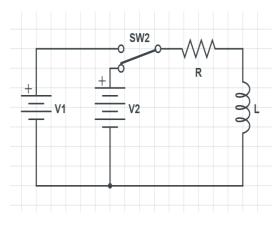


Current in the L-R circuit  $I(t) = \frac{V}{R} (1 - e^{-\frac{t}{\tau}})$  (Rising) Voltage across the resistor = I(t) R Voltage across the inductor =  $L \frac{d I(t)}{dt} = V e^{-\frac{t}{\tau}}$  (Decays) Current in the R-C circuit  $I(t) = \frac{V}{R} e^{-\frac{t}{\tau}}$ Voltage across the resistor = I(t) R Voltage across the capacitor =  $C \int I(t) dt = V(1 - e^{-\frac{t}{\tau}})$ 

## <u> R-L Circuit – V-I Graphs</u>







The switch moves from  $V_2$  to  $V_1$  at t=0

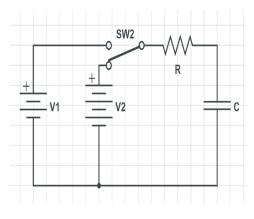
Current in the L-R circuit

$$I(t) = \frac{V_1}{R} - \frac{(V_1 - V_2)}{R} e^{-\frac{t}{\tau}}$$

Voltage across inductor

$$V(t) = (V_1 - V_2) e^{-\frac{t}{\tau}}$$

#### **R-C circuit**



Voltage across Capacitor V(t) = V<sub>1</sub> - (V<sub>1</sub> - V<sub>2</sub>)  $e^{-\frac{t}{\tau}}$ Current in the circuit

$$I(t) = \frac{(V_1 - V_2)}{R} e^{-\frac{t}{\tau}}$$

In general,

For rising exponentials,

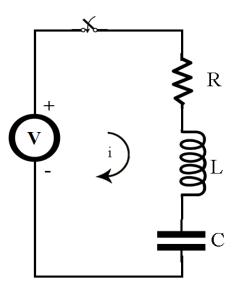
F(t) = Final value – (Final value – Initial value)  $e^{-\frac{t}{\tau}}$ If initial value is zero, F(t) = Final value (1 -  $e^{-\frac{t}{\tau}}$ )

For decaying exponentials,

F(t) = Final value + (Initial value – Final value)  $e^{-\frac{t}{\tau}}$ If final value is zero, F(t) = (Initial value)  $e^{-\frac{t}{\tau}}$ 

## **TOPIC 3** $\rightarrow$ **DC** Transients in R-L-C circuits

TOPIC 3.1  $\rightarrow$  Series R-L-C circuit



When a DC voltage is given to the R-L-C series circuit, the current in the circuit neither exponentially rises nor falls as both the L and C elements are involved.

$$\mathbf{V} = \mathbf{I}(\mathbf{t}) \mathbf{R} + \mathbf{L} \frac{\mathrm{d} \mathbf{I}(\mathbf{t})}{\mathrm{d}\mathbf{t}} \frac{1}{\mathrm{C}} \int \mathbf{I}(\mathbf{t}) \mathrm{d}\mathbf{t}$$
$$\frac{\mathrm{d} \mathbf{V}(\mathbf{t})}{\mathrm{d}\mathbf{t}} = \mathbf{0} = \mathbf{R} \frac{\mathrm{d} \mathbf{I}(\mathbf{t})}{\mathrm{d}\mathbf{t}} + \mathbf{L} \frac{\mathrm{d}^{2}\mathbf{I}(\mathbf{t})}{\mathrm{d}\mathbf{t}^{2}} + \frac{1}{\mathrm{C}} \mathbf{I}(\mathbf{t})$$
$$\frac{\mathrm{d}^{2}\mathbf{I}(\mathbf{t})}{\mathrm{d}\mathbf{t}^{2}} + \frac{\mathrm{R}}{\mathrm{L}} \frac{\mathrm{d} \mathbf{I}(\mathbf{t})}{\mathrm{d}\mathbf{t}} + \frac{1}{\mathrm{LC}} \mathbf{I}(\mathbf{t}) = \mathbf{0}$$
$$\mathbf{D}^{2} + \frac{\mathrm{R}}{\mathrm{L}} \mathbf{D} + \frac{1}{\mathrm{LC}} = \mathbf{0}$$

#### SURESH VSR

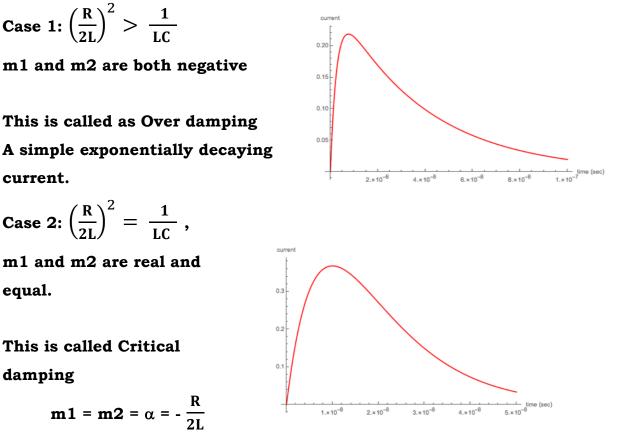
Solving this differential equation,

$$\mathbf{D} = \frac{-\mathbf{R}}{2 \mathbf{L}} \pm \sqrt{\left(\frac{\mathbf{R}}{2\mathbf{L}}\right)^2 - \frac{1}{\mathbf{LC}}} = \mathbf{m1} \text{ and } \mathbf{m2}$$

The final solution of the equation is

$$\mathbf{I(t)} = \mathbf{C}_1 \ \mathbf{e}^{\mathrm{m1t}} + \mathbf{C}_2 \ \mathbf{e}^{\mathrm{m2t}}$$

The values of m1 and m2 determine the nature of exponentials,



The current equation is  $I(t) = (C_1 + C_2 t) e^{\alpha t}$ 

A rising linear function and falling exponential function.

Case 3:  $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ , m1 and m2 are both complex

This is called Under damping

An exponentially decaying Harmonic current.

 $m1 = \alpha + j\omega$  and  $m2 = \alpha - j\omega$ 

The current equation is I(t) =  $C_1 e^{\alpha t} e^{-j\omega t} + C_2 e^{\alpha t} e^{+j\omega t}$ 

= (  $C_1 \cos \omega t + C_2 \sin \omega t$ )  $e^{\alpha t}$ 

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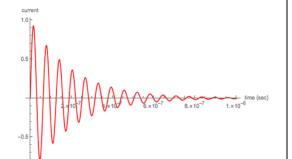
**NETWORK THEORY** 

#### SURESH VSR

Where 
$$\alpha = -\frac{R}{2L}$$
 or damping coefficient

and 
$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

Time Constant of the decay =  $\frac{1}{\alpha} = \frac{2L}{R}$ 



Case 4: R= 0, m1 and m2 are imaginary and equal. This is called as Undamped Harmonic A simple harmonic oscillations or LC circuit oscillations.

The current equation is  $I(t) = C_1 e^{-j\omega t} + C_2 e^{+j\omega t}$ 

Where 
$$\alpha = \mathbf{0}$$
 and  $\omega = \frac{1}{\sqrt{LC}}$ 

Damping Ratio

The ratio of  $\frac{\alpha}{\omega} = \xi$  is called as Damping Ratio  $\xi = \frac{R}{2L\frac{1}{\sqrt{LC}}} = \frac{R}{2}\sqrt{\frac{C}{L}}$ 

This is a measure of attenuation to phase shift nature

- $\xi > 1 \rightarrow \text{Over damped} \qquad \qquad \xi < 1 \rightarrow \text{Under damped}$
- $\xi = 1 \rightarrow Critically damped \quad \xi = 0 \rightarrow Undamped oscillations$

TOPIC 3.2  $\rightarrow$  Shunt R-L-C circuit

When a current source is applied to a shunt R-L-C circuit,

The voltage transient is as a duality of the series R-L-C circuit.

Damping Coefficient 
$$\alpha = \frac{1}{2 \text{ R C}}$$
,  $\omega = \frac{1}{\sqrt{\text{LC}}}$   
Damping Ratio  $= \frac{1}{2 \text{ R}} \sqrt{\frac{\text{L}}{\text{C}}}$ 



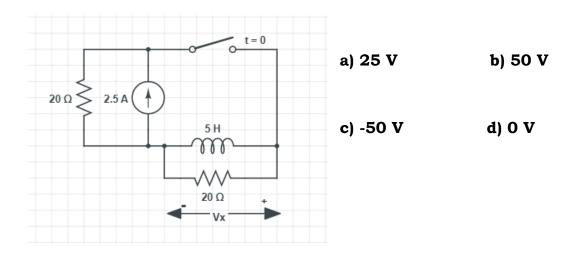
# **TRANSIENT ANALYSIS**

**WORK BOOK QUESTIONS** 

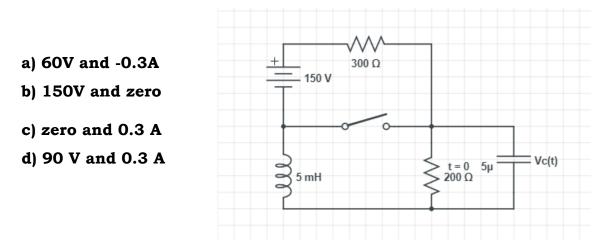
# **WORKBOOK QUESTIONS**

**TOPIC** 1.1  $\rightarrow$  L-C behavior after switching

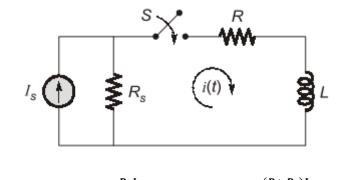
# Q1. The switch was closed for a long time before opening at t = 0. The voltage Vx at $t=0^+$ is



Q2. After keeping it open for a long time, the switch 'S' in the circuit shown in the given figure is closed at t=0. The capacitor voltage Vc(0<sup>+</sup>) and inductor current  $i_L(0^+)$  will be



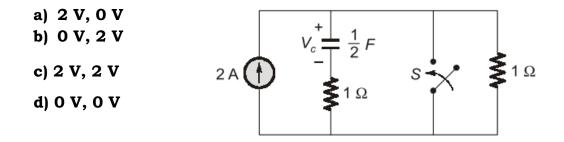
Q3. In the following circuit, the switch S is closed at t = 0. The rate of change of current  $\frac{di}{dt}$  at t=0<sup>+</sup> is given by



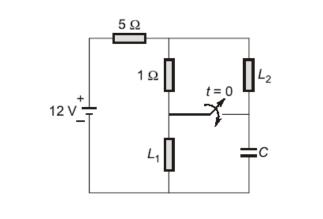
a) 0 b)  $\frac{R_s I_s}{L}$  c)  $\frac{(R+R_s)I_s}{L}$  d)  $\infty$ 

Q4. The circuit shown in the given figure is in steady- state with switch 'S' open. The switch is closed at t = 0.

The values of  $V_c(0)^+$  and  $V_c(\infty)$  will be respectively.



Q5. The circuit shown below is in steady-state with the switch open. At t = 0, the switch is closed. What is the current through the 1 ohm resistor at t=0<sup>+</sup>?



A) 0

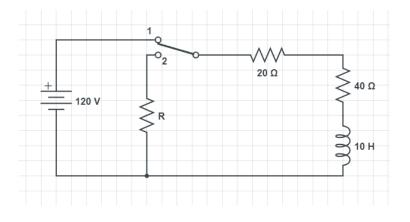
B) 1.33 A





Q6. A coil of inductance 10 H and resistance 40  $\Omega$  is connected as shown in the figure. After the switch S has been in contact with point I for a very long time, it is moved to point 2 at, t=0.

What is the inductor current after switching at t=0+?



Q7. Given at  $t = 0^+$ , the voltage across the coil is 120V, the value of resistance R is

**a)**  $0 \Omega$  **b)**  $20 \Omega$  **c)**  $40 \Omega$  **d)**  $60 \Omega$ 

Q8. For the value of R obtained in Q9, the time taken for 95% of the stored energy to be dissipated is close to

a) 0.10 sec b) 0.15 sec c) 0.50 sec d) 1.0 sec

Q9. In the circuit, steady state is reached with switch open.

At t = 0, switch is closed then  $V_a(0^+)$  is \_\_\_\_V

a) 2V b) 3V c) 5V d) 8V

5 V - 2 F

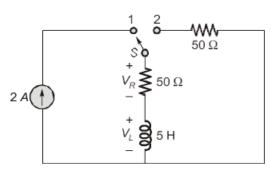
**GATE PRO** 105

SURESH VSR

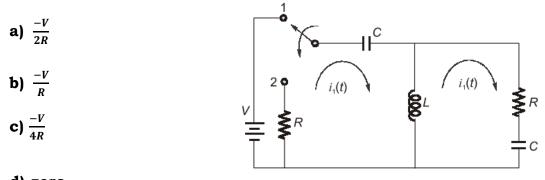
Q10. In the circuit shown, switch S is kept at position 1 for a long time.

Then at t = 0 the switch is transferred to position 2.

The voltage across inductor at t =  $0^+$  is \_\_\_\_V

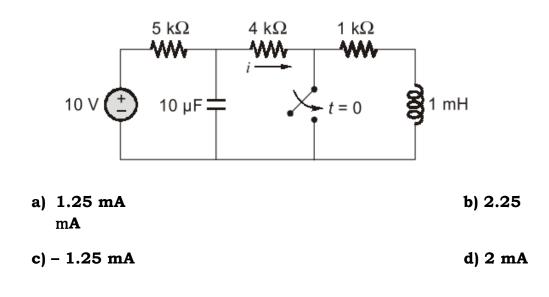


#### **Q11.** At $t = 0^+$ , the current $i_1$ is

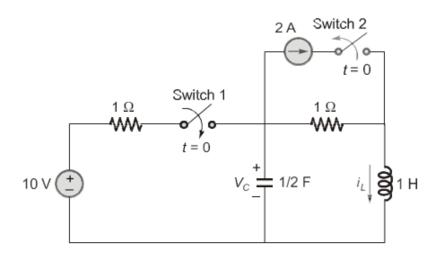


d) zero

Q12. In the figure shown, the ideal switch has been open for a long time. If it is closed at t = 0, then the magnitude of the current through the 4 k $\Omega$  resistor at t = 0<sup>+</sup> is



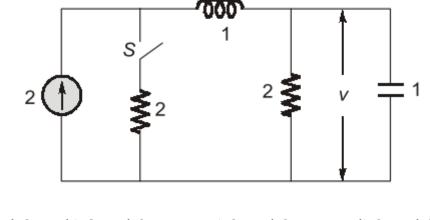
Q13. Assume the switch 1 has been opened and switch-2 has closed for a long time, and steady – state condition prevails at  $t = 0^{-}$ .



Then which of the following are correct?

a)  $\frac{dV_c(0^+)}{dt} = 16 \text{ V/s}$  b)  $V_c(0^+) = 2 \text{ V}$  c)  $\frac{di_L(0^+)}{dt} = 2 \text{ A/s} \text{ d})i_L(0^+) = 0 \text{ A}$ 

Q14. the circuit as shown below is in the steady state. The switch S is closed at t = 0. What are the values of v and  $\frac{dv}{dt}$  at t = 0<sup>+</sup>?



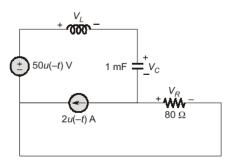
a) 0 and 4 b) 4 and 0 c) 2 and 0 d) 0 and 2

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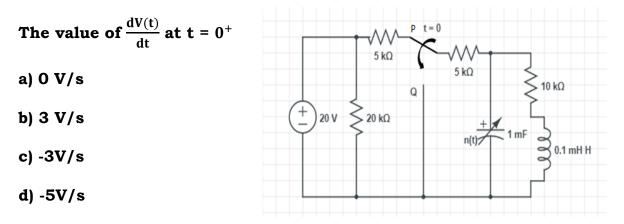
**NETWORK THEORY** 

Q15. In the circuit shown below,  $V_{C}(0^{+})$  is

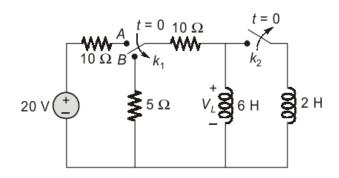
- a) 50 V b) 210 V
- c) 160 V d) -50 V



Q16. The switch in the circuit in the figure is in position P for a long time and then moved to position Q at time t=0



Q17. Consider the circuit given below,



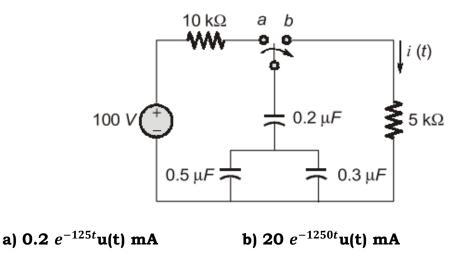
The switch  $k_1$  is kept at position A and switch  $k_2$  was closed for a long time. At t = 0 switch  $k_1$ , is moved to position B and  $k_2$  is opened. The voltage  $V_L$  across 6 H inductor at t = 0<sup>+</sup> is

a)5 V b) 3 V c) - 3.75 V d) - 4.75 V

**TOPIC 2**  $\rightarrow$  Exponential Equations in R-L and R-C Circuits

Q18. The switch in the circuit shown was on position a for a long time, and is moved to position b at time t = 0.

The current i(t) for t > 0 given by



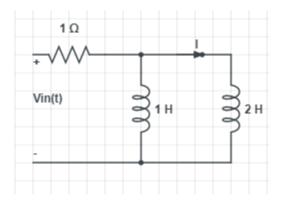
c) 0.2  $e^{-1250t}$ u(t) mA d) 20  $e^{-1000t}$ u(t) mA

Q19. In the circuit shown the voltage  $V_{IN}$  (t) is described by:

$$V_{IN}(t) = \begin{cases} 0, & \text{for } t < 0\\ 15V & \text{for } t \ge 0 \end{cases}$$

Where t is seconds.

The time (in seconds) at which the current 1 in the circuit will reach the value 2 Amperes is\_\_\_\_\_

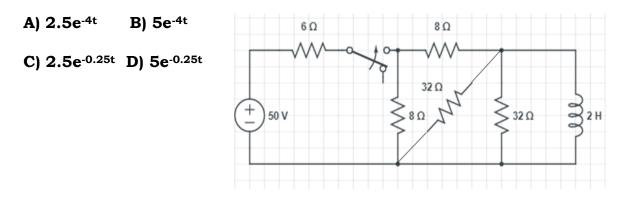


Q20. The switch in the figure below was closed for a long time. It is opened at t = 0. The current in the inductor of 2H for t $\geq$ 0, is

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**NETWORK THEORY** 

SURESH VSR



Q21. In the circuit shown below a step input voltage of magnitude

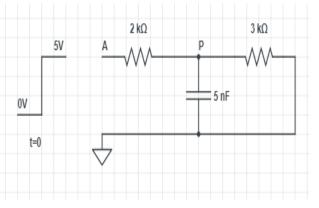
5 V in applied at node A at time t=0.

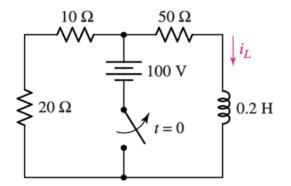
If the capacitor has no charge for the voltage at node p at t = 6  $\mu$ s is \_\_\_\_\_ V.

(Answer should be rounded off to two decimal places)

Q22. If the switch has been closed for long time and the switched is opened at t = 0. Find the time t where

I(t) = 0.5 I(t = 0) in the inductor

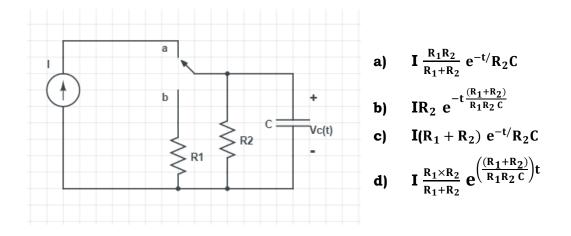




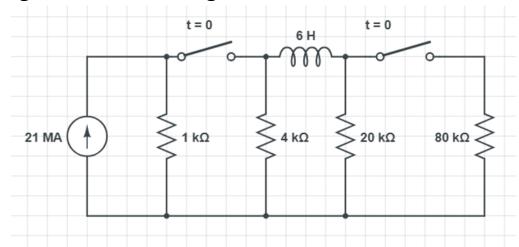
Q23. In the figure shown below switch is kept at position 'a' before moving to position 'b' at t =0. When switch is moved to position "b' voltage across capacitor V(t) is given by **GATEPRO** 

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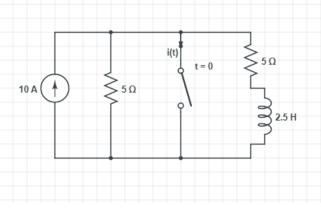
Q24. In the circuit below, both switches are open at t = 0 after having been closed for a long time.



After what time the energy dissipated in the 4 k $\Omega$  resistor will be 10 % of the initial energy stored in the inductor (in  $\mu$  sec)?

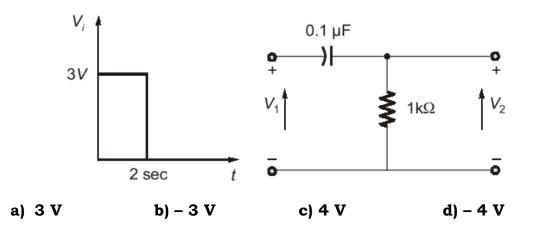
Q25. The switch in the circuit, shown in the figure, was open for a long time and is closed at t=0.

The current i(t) (in ampere) at t = 0.5 seconds is

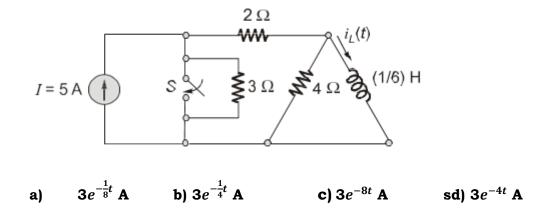


SURESH VSR

Q26. A square pulse of 3 volts amplitude is applied to C – R circuit. The capacitor is initially uncharged. The output  $V_2$  at time t = 2 sec is



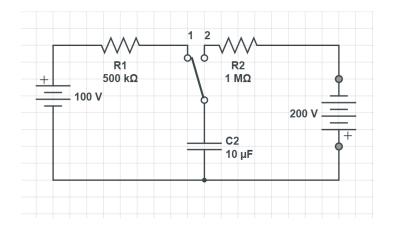
Q27. In the circuit, the value of  $i_L(t)$  at t = 0<sup>+</sup> after switch 'S' is closed, is (Assuming steady state condition prevailing before switching)



**TOPIC 2.3**  $\rightarrow$  Circuits with one active state to another Active State

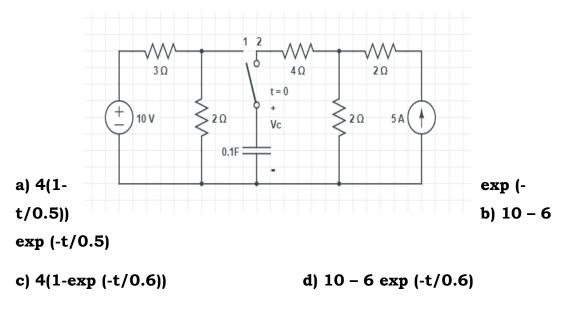
Q28. If the switch is moved from 1 to 2 positions at t = 0.

Find the time t where V(t) = 150 V in the capacitor

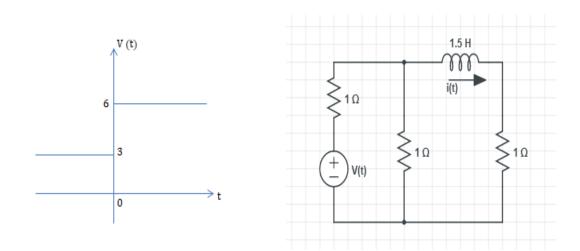


Q29. The switch has been in position 1 for a long time and abruptly changes to position 2 at t=0.

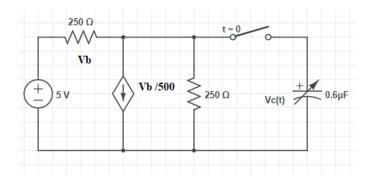
If time t is in seconds, the capacitor voltage  $V_{\rm C}$  (in volts) for t>0 is given by



Q30. The voltage v (t) shown below is applied to the given circuit. The value of current i(t) at t = 1s, in ampere is\_\_\_\_\_

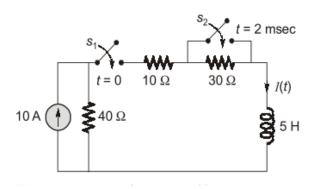


Q31. In the circuit shown in the figure, the switch is closed at time t = 0, while the capacitor is initially charged to -5V



The time after which the voltage across the capacitor becomes zero (rounded off to three decimal places) is\_\_\_\_\_ms

Q32. The switch  $S_1$  is closed at t = 0 and switch  $S_2$  closed at t = 2 msec



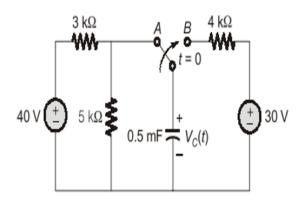
The magnitude of voltage v(t) across the inductor at t = 200mS is \_\_\_\_V

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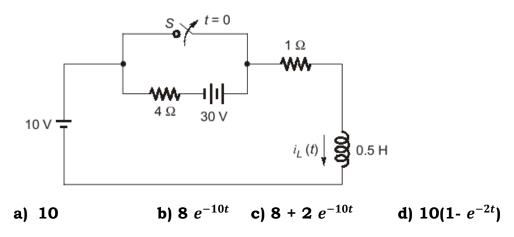
#### **NETWORK THEORY**

Q33. In the circuit shown in figure below; The switch has been in position A for a long time. At t = 0, the switch is moved to B. then, the capacitor voltage  $V_c(t)$  for t > 0 is

a) 
$$V_C(t) = (24 - 6e^{-2t})V$$
  
b)  $V_C(t) = (30 - 15e^{-0.5t})V$   
c)  $V_C(t) = (6 - 6e^{-2t})V$   
d)  $V_C(t) = (30 - 5e^{-0.5t})V$ 



Q34. In the circuit, switch 's' is in the closed position for a very long time. If the switch is opened at time t = 0, then  $i_L(t)$ , for  $t \ge 0$  is



Q35. In the circuit shown below, the switch is moved from position A to B at time t = 0. The current I through the inductor satisfies the following conditions.

1. 2. 3.	I(0) = - 8A dI/dt (t = 0) = 3 A/s I(∞) = 4 A then R =	$3 \Omega$
a)	0.5 ohm b)2.0 ohm	
C)	4.0 ohm. d)12 ohm	

# **NETWORK THEORY**

# **TRANSIENT ANALYSIS**

HINTS AND KEY - WORKBOOK

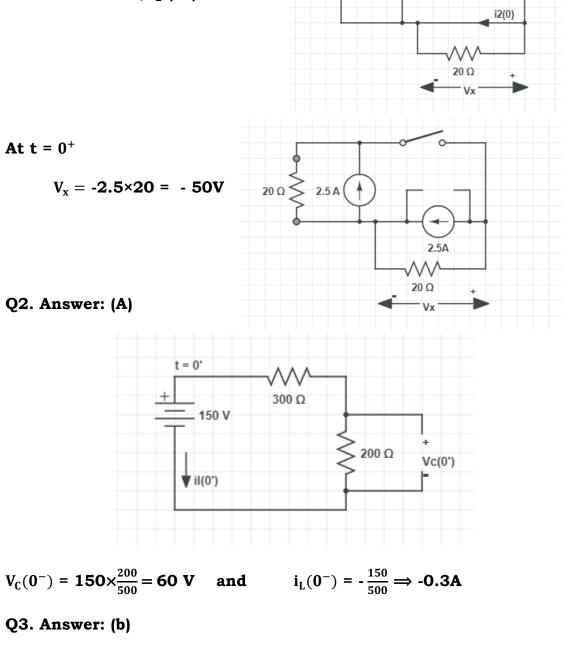
## Key & Hints CLASS-ROOM PRACTICE QUESTIONS

 $20 \Omega \ge 2.5 A$ 

TOPIC  $1.1 \rightarrow$  L-C behavior after switching

Q1. Answer: (c)

At  $t = 0^-$ , Inductor acts as a short circuit so ,  $i_2 (0^-) = 2.5A$ 

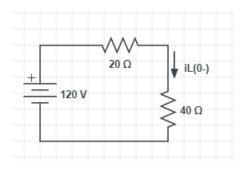


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- Q4. Answer: (a)
- Q5. Answer: (D)

Q6. Answer: (c)

At  $t = 0^-$ , Inductor is replaced by short circuit so

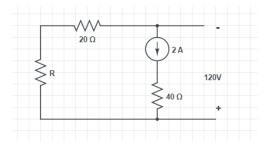


 $i_L(0^-) = 120/60 = 2A$  also  $i_L(0^+) = 2A$ 

Q7. Answer: (d)

**At t = 0^+** 

Solving 120 = 2(R) + 2(20),  $R = 40\Omega$ 

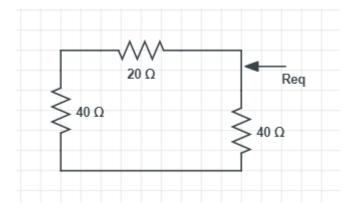


Q8. Answer: (b)

 $i(\infty) = OA$  (inductor is source free circuit)

 $\tau = 4 \text{ Req}$ 

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Req = 40+40+20 = 100 $\Omega$ , Leq = 10H and  $\tau = \frac{10}{100} = 0.1$ sec

 $i(t) = i(\infty) + (i(0^+) - i(\infty)) e^{-t/\tau} = 0 + (2-0) e^{-t/0.1} = 2e^{-10t}$ , At=0, i(t) =2A

Energy stored in inductor  $W_L = \frac{1}{2} \times L \times I_L^2(0^-) = \frac{1}{2} \times 10 \times 2^2 = 20$ Joules

Inductor looses this energy in this resistance,

$$\mathbf{W}_{\mathbf{R}} = \int_{0}^{t} \mathbf{i}(t)^{2} \mathbf{R} \, dt = 0.95 \times 20 = \int_{0}^{t} (2e^{-10t})^{2} 100 \, dt$$
,  $t = 0.15s$ 

Q9. Answer: (b)

Q10. Answer: (200V)

Q11. Answer: (A)

Q12. Answer: (a)

Q13. Answer: (a,b,c,d) All 4 options right.

Q14. Answer: (b)

Replace the Inductor with 2A current source,

Capacitor with 4V source,

The 2A from inductor flows in 2  $\Omega$  on right side and I = 0 in 4V source.

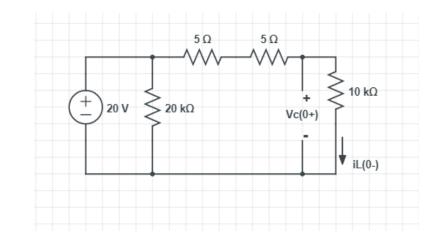
### dV/dt = 0 and V = 4

Q15. Answer: (B)

Q16. Answer: (c)

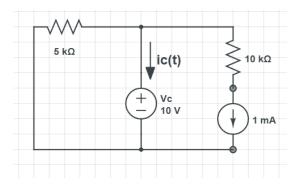
At  $t = 0^-$  (with source), Inductor is a short circuit

Capacitor with source is an Open circuit,



$$V_{C}(0^{-}) = 20 \times \frac{10}{20} = 10V$$
 and  $i_{L}(0^{-}) = \frac{20}{10+10} = 1mA$ 

At  $t = 0^+$ ,



 $i_c(0+) + 10/5k + 1 mA = 0$ ,  $i_c(0+) = -3mA$ 

$$C \frac{dV}{dt} = -3mA$$
,  $\frac{dV}{dt} = = 3 V/second$ 

Q17. Answer: (5V)

Before switching, Current flowing in the 10 $\Omega$  elements = 1A

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120			

This divides as 2/3 into 2H and 1/3 in 6H,

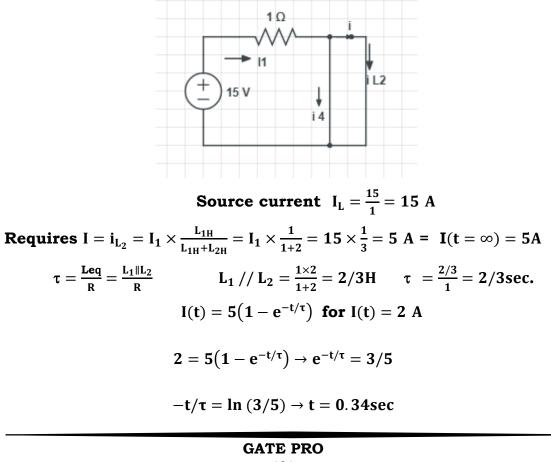
After switching, 1/3 Amps discharges through 10 and 5  $\Omega$  series, The voltage across inductor = 5V

### **TOPIC 2** $\rightarrow$ **Exponential Equations in R-L - R-C Circuits**

Q18. Answer: (B)

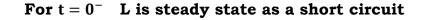
Q19.  $V_{in}(t) = 0 \text{ at } t < 0$ At  $t = 0^-$ , Circuit is source less  $i_{L_1}(0^-) = 0 \text{ and } i_{L_2}(0^-) = 0$ For  $t = 0^+$ ;  $i_{i_1}(0^+) = i_{L_1}(0^-) = 0$  and  $i_{L_2}(0^+) = i_{L_2}(0^-) = 0$  $i(0^+) = 0$ 

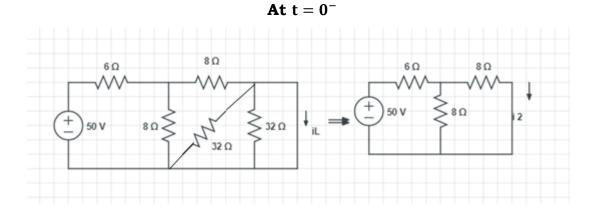
For  $t \rightarrow \infty$  both 1H and 2H will be in steady state as short circuits



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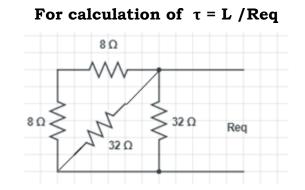
Q20. Answer (c)





 $\label{eq:Fort} \textbf{For} \; t = 0^+, \qquad i_{i_1}(0^+) = i_{L_1}(0^-) = 2.\,5A$ 

For  $t \to \infty$ , L is without source  $I_L(\infty) = OA$ 

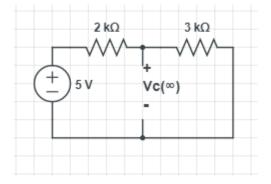


Req = (8+8) || (32||32) = 16||16 = 8 $\Omega$   $\tau = \frac{2}{8} = \frac{1}{4}$ I(t) = 2.5 e<sup>-4t</sup>

Q21. Answer : 1.896

$$\begin{split} &V_c(0^-) = 0 \ V \\ &V_c(0^+) = 0 \ V \end{split}$$

At  $t \to \infty$ , capacitor acts as a open circuit



$$V_{C}(\infty) = 5 \times \frac{3}{5} = 3 V$$
  $\tau = (\frac{2 \times 3}{2 + 3}) \times 5 \mu S = 6 \mu S$ 

**Expression for**  $V_c(t)$ 

 $V_{C}(t) = V_{C}(\infty) + (V_{C}(0^{-}) - V_{C}(\infty)) e^{-t/\tau} = V_{C}(t) 3 + (0-3) e^{-t/\tau} = 1.896 V$ 

Q22. Initial current after opening the switch at  $t = 0^+$  is,

100/50 = 2 Amps as inductor was short circuit at t =0-

This 2A discharges through the 10, 20 and 50 $\Omega$  series combination,

Time constant =L/R = 0.2 / 80 = 2.5 mSec I(t) =  $2e^{-t\frac{R}{L}} = 1$ Solving, ln2 = t /2.5 mS , t = 1.73 mS

Q23. Answer (b)

Voltage before switching  $ON = V_C(t = 0) = V_{R2} = I R_2$ 

This voltage discharges through the shunt  $R_1$  and  $R_2$  combination with time constant Req x C.

Q24. Answer ()

The current flowing in the Inductor before switching = 1A

This discharges through 24K resistors,

10% of initial energy is dissipated in 4K then 50% in 24K = 60% of initial value of inductor energy is dissipated.

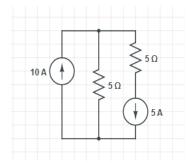
40% of initial energy is left out.

Energy ( time ) = Initial energy  $e^{-2t/\tau}$ 

 $\tau = 6/24000 = 0.25 \text{ mS}$  and t =  $\ln(2.5) / 8000 \text{ S}$ 

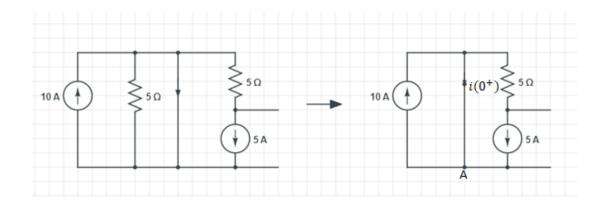
Q25. Answer: 8.16

For  $t = 0^-$ , Inductor is in steady state and short circuit,



At  $i_L(0^-) = 5A$ ,  $i_L(0^+) = i_L(0^-) = 5A$ 

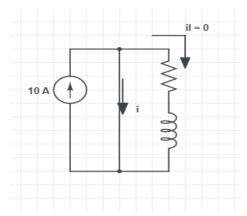
For  $t = 0^+$ 



From KCL at node A ,  $i(0^-) = 10 - 5 = 5$ Amps

Now due to the short circuit path available, 10A from current source does not enter the inductor branch.

Inductor discharges through the short circuit and 5  $\Omega$ 



i(∞) = 10A, So i(t) = 10 + (5-10)e<sup>-tL/R</sup>  $\tau = \frac{L}{R} = \frac{2.5}{5} = \frac{1}{2}$ 

i(t) = 10 - 5 $e^{-2t}$  and i(t =0.5 sec) = 10-5  $e^{-2\times\frac{1}{2}}$  = 10-5 $e^{-1}$  = 8.16A Q26. Answer (b) Q27. Answer (c)

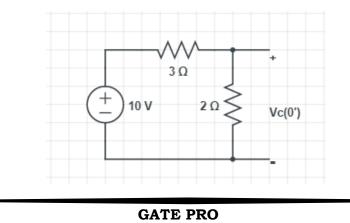
**TOPIC 2.3**  $\rightarrow$  Circuits with one active state to another Active State

Q28. The voltage changes from 100 to -200 V with exponential function of time as,

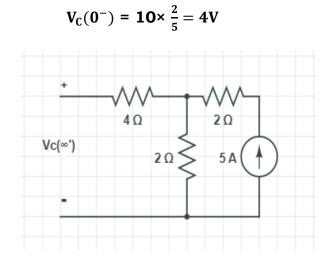
$$V_{C}(t) = -200 + (100 - (-200)) e^{-\frac{t}{RC}} = -150$$
  
 $300e^{-\frac{t}{RC}} = 50$ ,  $t = RC \ln(6) = 10\ln6 = 17.9$ seconds

Q29. Answer: (d)

At  $t = 0^-$ , Capacitor is act as open circuit



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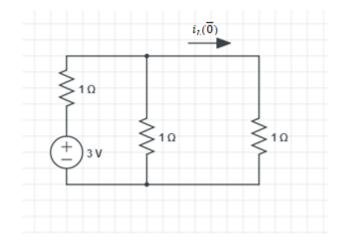
At t = 
$$\infty$$

$$W_{C}(\infty^{-}) = 5 \times 2 = 10V$$
,  $\tau = RC = 6 \times 0.1 = 0.6$ 

$$V_{\rm C}(t)$$
 = 10 – 6 e<sup>-t/0.6</sup>

### Q30. Answer: 1.632

### At t = $0^-$ , Inductor acts as short circuit

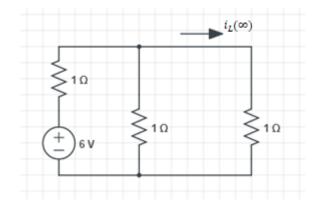


$$i_L(0^-) = \frac{3}{1.5} \times \frac{1}{1+1} = 1$$
 Amps

### At $\mathbf{t} = \infty$

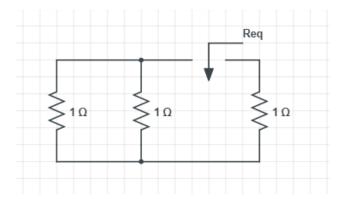
**NETWORK THEORY** 

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$$i_L(\infty) = \frac{6}{1.5} \times \frac{1}{1+1} = 2A$$

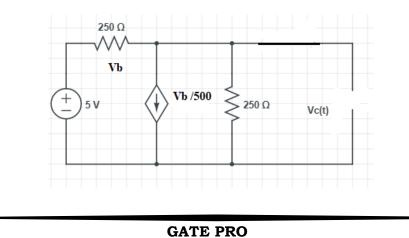
For Time constant



$$\begin{split} \text{Req} = (1 \text{ parallel } 1 \text{ }) + 1 &= 1.5\Omega \text{ , } \quad \text{L} = 1.5 \text{ H } \quad \tau = \frac{1.5}{1.5} = 1 \text{ second } \text{ ,} \\ i_L(t) &= i_L(\infty) + (i_L(0) - i_L(\infty)) \text{ } e^{-t/\tau} = 1 + (1-2)e^{-t} = 1.632 \text{ A} \end{split}$$

#### Q31. Answer: 0.1386





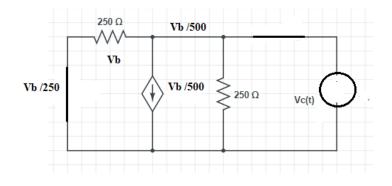
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$$V_{\rm c}(\infty) = 250.\frac{V_{\rm b}}{500} = \frac{V_{\rm b}}{2}$$

Using KVL, 5-V<sub>b</sub> - 
$$\frac{V_b}{2} = 0$$
,  $V_b = \frac{10}{3}$  V  $V_c(\infty) = \frac{V_b}{2} = \frac{5}{3}$  V

Time constant ( $\tau$ ) =  $R_{eq}$  C



Apply level for outside loop V = -V<sub>R</sub> , V = 250  $\left(I + \frac{V_R}{500}\right)$ 

Req =  $\frac{v}{1}$  = 166.67 $\Omega$ ,  $\tau$  = (166.67)(0.6 $\mu$ ) =1000  $\mu$  Seconds  $V_c(t) = \frac{5}{3} + (-5 - \frac{5}{3}) e^{-t/100} = 0 = \frac{5}{3} - \frac{20}{3} e^{-t/100}$ , t = 0.1386 ms

Q32. Answer: (54.14 V)

Q33. Answer: (d)

Q34. Answer: (c)

Q35. Answer: (C)

# **NETWORK THEORY**

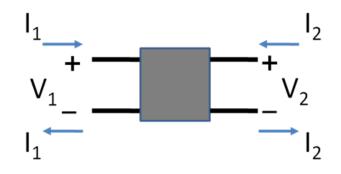
# **TWO PORT NETWORKS**

# **THEORY – SHORT NOTES**

### **TOPIC** 1 $\rightarrow$ Introduction

A network across any branch (1 port) can be reduced by using Thevenin's or Norton's theorem.

Similarly, any network across it's two ports can be reduced with simpler elements like voltage sources and current sources. The simplified network to be used depends on the network's nature and the dependent and independent port variables.



### **TOPIC** 1.1 $\rightarrow$ Z Parameters

If the port voltages depend on the currents in the port, Z parameters rightly define the network.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

 $Z_{11} = V_1/I_1$  when  $I_2 = 0$ 

= Input impedance at port1 when port2 is open circuited

 $Z_{22} = V_2/I_2$  when  $I_1 = 0$ 

= Output impedance at port2 when port1 is open circuited

$$Z_{12} = V_1 / I_2$$
 when  $I_1 = 0$ 

= Backward Trans-impedance when port1 is open circuited

$$Z_{21} = V_2/I_1$$
 when  $I_2 = 0$ 

= Forward Trans-impedance when port2 is open circuited

### **TOPIC 1.2** $\rightarrow$ **Y Parameters**

If the port currents depend on the voltages in the port,

Y parameters rightly define the network.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

 $Y_{11} = I_1/V_1$  when  $V_2 = 0$ 

= Input admittance at port1 when port2 is short circuited

$$Y_{22} = I_2/V_2$$
 when  $V_1 = 0$ 

= Output admittance at port2 when port1 is short circuited

$$Y_{12} = I_1/V_2$$
 when  $V_1 = 0$ 

= Backward Trans-admittance when port1 is short circuited

$$Y_{21} = I_2/V_1$$
 when  $V_2 = 0$ 

= Forward Trans-admittance when port2 is short circuited

**TOPIC** 1.3 $\rightarrow$  h (Hybrid) Parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$
  
 $I_2 = h_{21} I_1 + h_{22} V_2$ 

 $h_{11} = V_1 / I_1$  when  $V_2 = 0$ 

= Input impedance at port1 when port2 is short circuited

 $h_{22} = I_2/V_2$  when  $I_1 = 0$ 

= Output admittance at port2 when port1 is open circuited  $h_{12} = V_1/V_2$  when  $I_1 = 0$ 

= Reverse voltage gain when port1 is open circuited

 $h_{21} = I_2/I_1$  when  $V_2 = 0$ 

= Forward current gain when port2 is short circuited

**TOPIC 1.4** $\rightarrow$  g (Hybrid) Parameters

$$I_1 = g_{11} V_1 + g_{12} I_2$$
$$V_2 = g_{21} V_1 + g_{22} I_2$$

**TOPIC 1.5**  $\rightarrow$  **Transmission Parameters (A, B, C, D)** 

$$\mathbf{V}_1 = \mathbf{A} \ \mathbf{V}_2 - \mathbf{B} \ \mathbf{I}_2$$
$$\mathbf{I}_1 = \mathbf{C} \ \mathbf{V}_2 - \mathbf{D} \ \mathbf{I}_2$$

The minus sign signifies the port 2 current never enters into the network , rather it leaves the network into the load.

### **TOPIC** 1.6 $\rightarrow$ Inter- conversion between parameters

- 1. Write down the equations of both the parameters.
- 2. In the first set put the V or I as zero as desired in the second set of parameters to be converted.

Ex: Express h parameters in terms of Y parameters.

Y parameter equations,

Equation $1 \rightarrow$	$I_1 = Y_{11} V_1 + Y_{12} V_2$
Equation 2 $\rightarrow$	$I_2 = Y_{21} V_1 + Y_{22} V_2$

h parameter definitions,

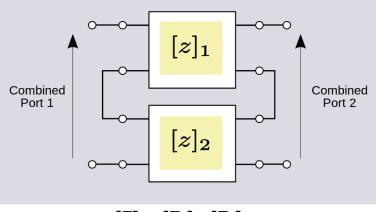
 $h_{11} = V_1/I_1$  when  $V_2 = 0$ , Use Equation 1 of Y parameters

 $h_{11} = 1/Y_{11}$ 

 $h_{22} = I_2/V_2$  when  $I_1 = 0$  Use Equation 1 with  $I_1 = 0$  in Equation 2  $h_{12} = V_1/V_2$  when  $I_1 = 0$  Use Equation 1 of Y parameters  $h_{21} = I_2/I_1$  when  $V_2 = 0$  Use Equation 1 with  $V_2 = 0$  in Equation 2

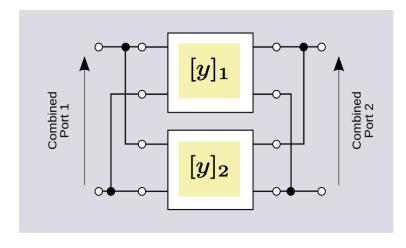
## **TOPIC 1.7** $\rightarrow$ Addition of 2 port networks parameters

Series addition of input and output voltages makes the combined port voltages as shown below, Z parameters of each port can be added to get the final port voltages.



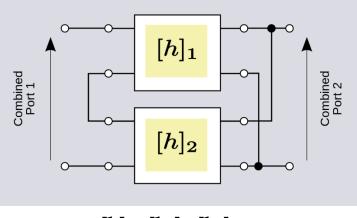
 $[\mathbf{Z}] = [\mathbf{Z}_1] + [\mathbf{Z}_2]$ 

Shunt addition of input and output currents makes the combined port voltages as shown below, Y parameters of each port can be added to get the final port voltages.



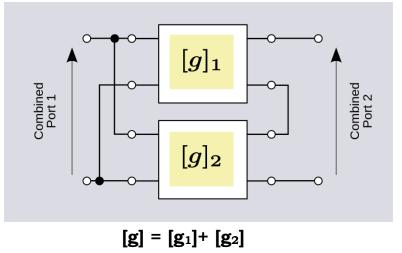
 $[\mathbf{Y}] = [\mathbf{Y}_1] + [\mathbf{Y}_2]$ 

### Input port voltages in series and Output port currents in shunt

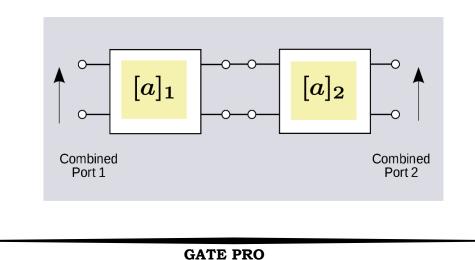


 $[h] = [h_1] + [h_2]$ 

### Input port currents in shunt and Output port voltages in series



Cascade connection of 2 port networks, Transmission parameters can be multiplied as,  $[T] = [T_1] \times [T_2]$ 



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### **TOPIC 1.8** $\rightarrow$ Symmetry and Reciprocity Conditions

**Reciprocity:** 

If the ratio of voltage at one port to the current at other port is same as the ratio with the positions of voltage and current are interchanged, then the network is said to be Reciprocal Two Port Network

It is possible when,

$$Z_{12} = Z_{21}$$
  
 $Y_{21} = Y_{12}$   
 $h_{12} = -h_{21}$   
[T] = AD - BC = 1

Symmetry:

If the input impedance is equal to the output impedance the network is said to be symmetrical Two Port Network

$$Z_{11} = Z_{22}$$
  
 $Y_{11} = Y_{22}$   
[ h ] = h<sub>11</sub> h<sub>22</sub> - h<sub>12</sub> h<sub>21</sub> = 1  
A = D

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# **NETWORK THEORY**

# **TWO PORT NETWORKS**

# **WORKBOOK QUESTIONS**

## **WORKBOOK QUESTIONS**

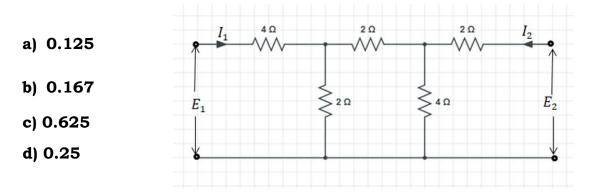
### **TOPIC** $1 \rightarrow$ Introduction

Q1. Find the Z, Y and T parameters of series resistance R as a 2 port network.

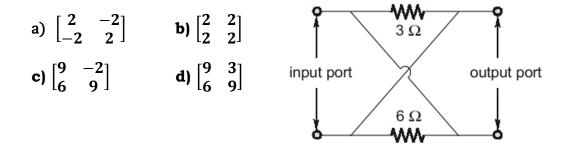
Q2. Find the Z, Y and T parameters of shunt resistance R as a 2 port network.

Q3. Find the Z parameters of a T network and Pi networks of three resistances R1, R2 and R3.

Q4. Among the h-parameter for a two-port network the value of  $h_{12}$  is

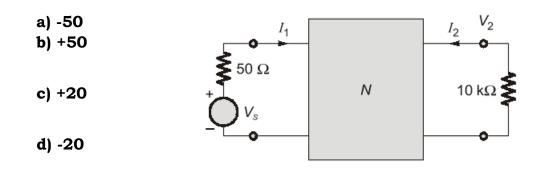


Q5. The Z-parameter matrix  $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$  for the two-port network is

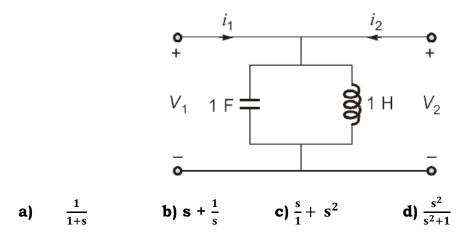


Q6. In the circuit shown, 2-port network N has

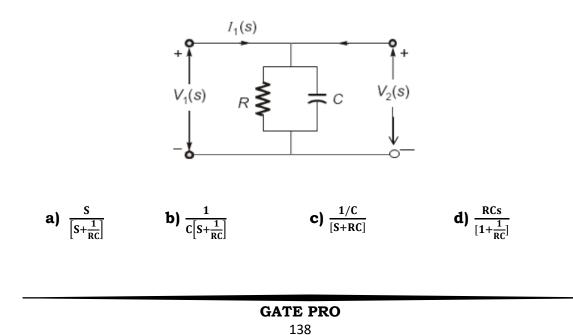
 $Z_{11}$  =  $10^3\Omega,$   $Z_{12}$  =  $10\Omega,$   $Z_{21}$  =  $-10^6\Omega$  and  $Z_{22}$  =  $10^4\Omega.$  The current gain  $\frac{l_2}{l_1}$  is



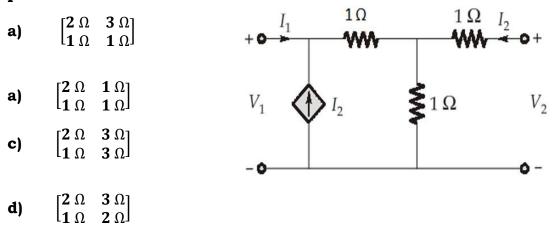
# Q7. In the parallel LC circuit shown in the figure below, the transmission line parameter C(s) will be equal to



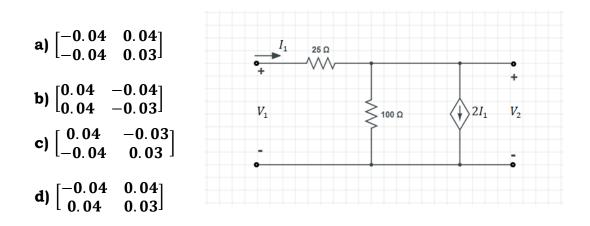
Q8. The value of  $Z_{21}(S)$  in the circuit shown below is



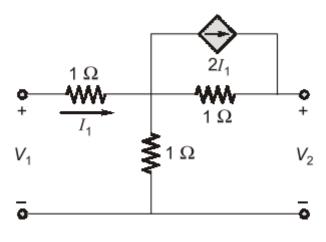
Q9. In the circuit shown in the figure below, the equivalent Zparameter matrix is



#### Q10. The Y-parameters of the network shown below are



Q11. In the two-port network shown, the  $h_{11}$  parameter (where,  $h_{11} = \frac{V_1}{I_i}$ , when  $V_2 = 0$ ) in ohms is \_\_\_\_\_ (up to 2 decimal place)



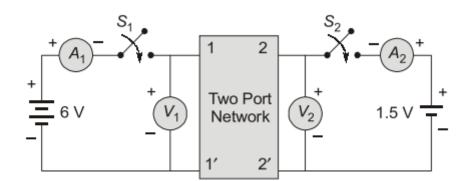
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Q12. A two-port network shown below is excited by external dc source. The voltage and current are measured with voltmeter  $V_1$ ,  $V_2$  and ammeters  $A_1$ ,  $A_2$  (all assumed to be ideal) as indicated. Under following to be ideal) as indicated. Under following switch conditions, the readings obtained are:

1)  $S_1$  - open,  $S_2$ - closed  $A_1$  = 0 A,  $V_1$  = 4.5 V,  $V_2$  = 1.5 V,  $A_2$  = 1 A

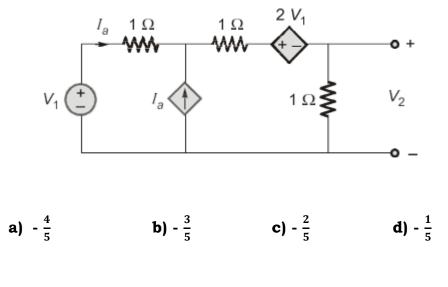
2)  $S_1$  - closed,  $S_2$ - open  $A_1$  = 4 A,  $V_1$  = 6 V,  $V_2$  = 6 V,  $A_2$  = 0 A



The Z- parameter matric for this network is

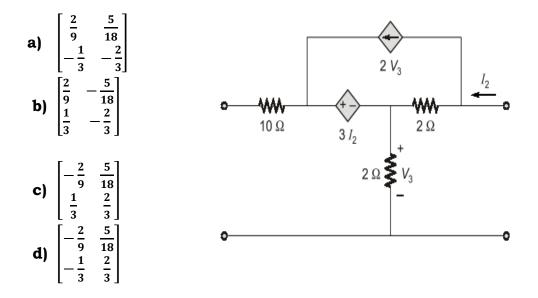
a) $\begin{bmatrix} 1.5 \\ 4.5 \end{bmatrix}$	1.5	b) [1.5	4.5
	1.5	1.5	4.5
c) $\begin{bmatrix} 1.5\\ 1.5 \end{bmatrix}$	4. 5 1. 5	d) $\begin{bmatrix} 4.5\\ 1.5 \end{bmatrix}$	1.5 4.5]

Q13. In the network contains resistors and controlled source  $G_{12}$  =  $\frac{V_2}{V_1}$ 

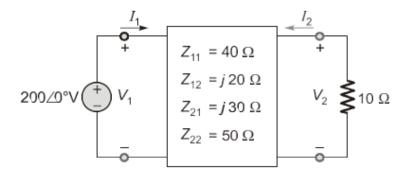


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Q14. In the circuit given below contains a voltage- controlled source and a current- controlled source. For the elements values specified, determine Y-parameters.

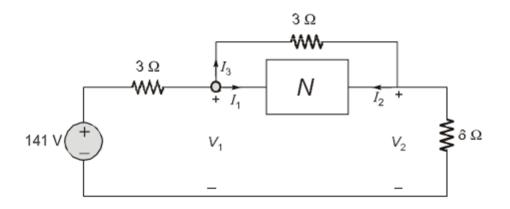


#### Q15. Consider the two port network given below



Then value of  $|I_1|$  is \_\_\_\_\_A

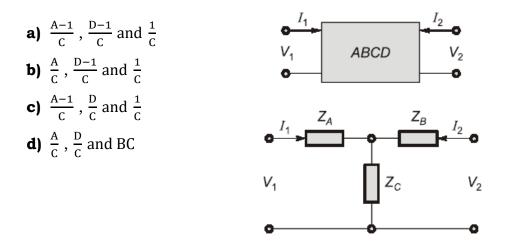
Q16. In the two port network N shown below in figure, if Z parameter matrix is  $\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$ , then current I<sub>1</sub> is \_\_\_\_\_A.



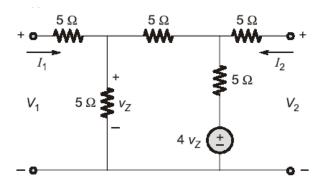
Q17. A two port network has a load of  $R_L$  across it's output port 2. Express it's input impedance in terms of its Transmission parameters ABCD and the load  $R_L$ .

**A.** 
$$\frac{A+BR}{C+DR}$$
 **B.**  $\frac{C+AR}{D+BR}$  **C.**  $\frac{A+DR}{C+BR}$  **D.**  $\frac{B+AR}{D+CR}$ 

Q18. In terms of the ABCD parameters of the network, express the impedance values  $Z_A$ ,  $Z_B$  and  $Z_C$  of the T network shown below.



Q19. The  $Z_{11}$  parameter of the network shown below is.....



### **TOPIC 1.6** $\rightarrow$ Inter- conversion between parameters

Q20. A 2 port network is given by the equations

$$V_1 = 60 I_1 + 20 I_2$$

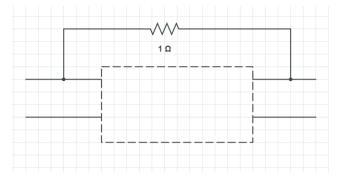
$$V_2 = 20 I_1 + 40 I_2$$

The ABCD parameters of the network are....

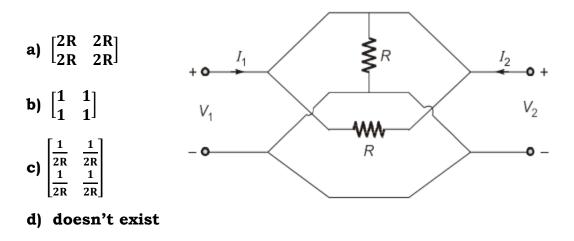
### **TOPIC 1.7** $\rightarrow$ Addition of 2 port networks parameters

Q21. The Y-parameter matrix of the block is shown below is  $\begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$ 

If a  $1\Omega$  resistor is connected as shown the new Y matrix is .....

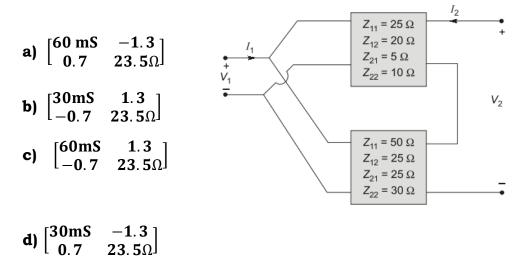


#### Q22. The Y-parameter matrix of the circuit shown below is



# Q23. Consider the parallel-series combination of two networks shown in the figure below:

#### The overall 'g' parameter for the circuit is equal to



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# **NETWORK THEORY**

# **TWO PORT NETWORKS**

# **KEY AND HINTS WORKBOOK**

**NETWORK THEORY** 

## **TOPIC 1** $\rightarrow$ Introduction

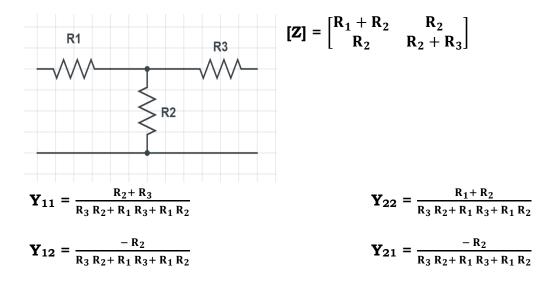
Q1. Answer:

$$\mathbf{Z} = \begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix} \qquad \mathbf{Y} = \begin{bmatrix} \frac{1}{R} & \frac{-1}{R} \\ \frac{-1}{R} & \frac{1}{R} \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}$$

Q2. Answer:

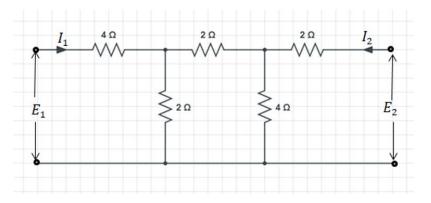
$$[\mathbf{Z}] = \begin{bmatrix} \mathbf{R} & \mathbf{R} \\ \mathbf{R} & \mathbf{R} \end{bmatrix} \qquad \mathbf{Y} = \begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix} \qquad [\mathbf{T}] = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \frac{1}{\mathbf{R}} & \mathbf{1} \end{bmatrix}$$

Q3. Answer:



$$\mathbf{h}_{11} = \frac{1}{Y_{11}}$$
  $\mathbf{h}_{12} = \frac{R_2}{R_2 + R_3}$   $\mathbf{h}_{22} = \frac{1}{Z_{22}}$   $\mathbf{h}_{21} = \frac{-R_2}{R_2 + R_3}$ 

Q4. Answer: (0.25)



 $\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} \Big|_{\mathbf{I}_1 = \mathbf{0}}$ 

$$\mathbf{E}_2 = \mathbf{V}_2 = \mathbf{4}\mathbf{I}_2$$

$$E_1 = V_1 = 2 \times \frac{l_2}{2} = I_2$$
  $h_{12} = 0.25$ 

- Q5. Answer : (B)
- Q6. Answer : (B)

$$V_1 = (10^3)I_1 + (10)I_2$$

 $V_2 = (-10^6) I_1 + (10^4) I_2$ 

### From the circuit diagram

$$V_2 = -(10k\Omega) I_2$$
  
 $-I_2 = -100I_1 + I_2$   
 $100I_1 = 2I_2$   
 $\frac{I_2}{I_1} = 50$ 

- Q7. Answer : (B)
- Q8. Answer : (B)

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Q9. Answer : (C)

Q10. Answer (B)

Q11. Answer : (0.5 Ohms)

Q12. Answer : (C)

Q13. Answer : (C)

Q14.Answer : (A)

Q15. Answer : (4 A)

Q16. Answer : (24 A)

Q17. Answer : (D)

$$Z_1 = \frac{V_1}{I_1} = \frac{B + AR_L}{D + CR_L}$$

Q18. Answer : (A)

 $Z_A = \frac{A-1}{c} \qquad \qquad Z_B = \frac{D-1}{c} \qquad \qquad Z_C = \frac{1}{c}$ 

Q19. Answer : ()

$$R_m = \frac{V_1}{I_1} = -5\Omega$$

### **TOPIC 1.6** $\rightarrow$ Inter- conversion between parameters

Q20. Answer : ()

$$[\mathbf{T}] = \begin{bmatrix} 3 & 100 \\ \frac{1}{20} & 2 \end{bmatrix}$$

## **TOPIC 1.7** $\rightarrow$ Addition of 2 port networks parameters

**Q21. Answer** :  $\begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$ 

Q22. Answer : (D), All the values are infinite.

Q23. Answer : (A)

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**NETWORK THEORY** 

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AIR 88 EE Meer Ejas Hussain



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